Journal
of
The Franklin Institute
Devoted to Science and the Mechanic Arts

Vol. 258 NOVEMBER, 1954 No. 5

SOME PROMINENT MEMBERS OF THE FRANKLIN INSTITUTE

1. SAMUEL VAUGHAN MERRICK, 1801–1870

BY

THOMAS COULSON[1]

Samuel Vaughan Merrick will be best known to our readers as the founder of The Franklin Institute, but it is not for this reason he has been chosen as the subject to open this series of sketches. He had other claims to eminence that should be placed on record before they become embalmed in time long since past. For he was distinguished not only as a successful man of business but, possessing that strength of character and clarity of judgment that mark a man for leadership, he was chosen by his fellow citizens to assume responsibility for communal and corporate enterprise on a large scale.

Born at Hallowell, Maine, on May 4th, 1801, he was the eldest son of John Merrick who had been trained in England for the Unitarian ministry. Concluding that he was unsuited for this calling, the elder Merrick had emigrated to the United States and, in the pleasant town of Hallowell, lived the life of a retired gentleman. The father of Samuel Merrick was a man of so many parts that a few words must be devoted to him, for he exercised a strong influence in molding the character of his eldest son. We have described him as living in the manner of a retired gentleman. It should be said, as one who has retired from active business but who retained an interest in the affairs of the community. He served at different times as selectman, surveyor of highways, and overseer of the poor. His interest in education led him to become one of the trustees of the Hallowell Academy and a member of the Board of Overseers of Bowdoin College from 1805 to 1851. His friend, Professor Goodwin, provost of the University of Pennsylvania, wrote a *Memoir* of this versatile man, describing his manifold interests, which ranged broadly through music, literature, and the sciences. Goodwin

---

[1] Director of Museum Research, The Franklin Institute, Philadelphia, Pa.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

said of him: "His was a noble mind, a noble heart, and a noble life. His faults were few, his enemies were none."

Very much the same could have been written about his son, for some of the parent's admirable qualities were transmitted to him and were to find expression on a larger stage than the little town of Hallowell offered him.

The environment during youth and adolescence plays an important part in the molding of character. We have outlined something of the domestic influence in which the young Merrick grew up, but we must not omit reference to his natural surroundings, which were of exceptional beauty. Those who know the Kennebec River will tell you there is no more beautiful view than that from the top of Powder House Hill in Hallowell. We may suppose, therefore, that the scenes of natural beauty were not wasted on an intelligent boy under the tutelage of a father who had a keen eye for beauty and the mind to interpret it to a receptive pupil.

These, then, were the influences under which Samuel Merrick's character were formed. He was never an obtrusive man although he was in the forefront of many movements. His high reputation was based entirely on more substantial qualities than those which feed on publicity and adulation. Those who knew him best admired him for his stability, his energy, and his practical wisdom. Most men can be fitted into one of two classes, the thinkers and the men of action. Rarely does nature endow a man with a combination of qualities that make him both thinker and actor, but Merrick was such a man.

New England is not richly endowed with natural resources but Nature has tried to make redress by bestowing upon its native sons a sense of resourcefulness. This, too, was a marked feature in Merrick's character.

Residence in a small town in Maine did not offer any sentimental attraction to a boy whose family traditions were not rooted in its soil, nor any material prospects to one who had ambition. Of course, there was ship-building and Bath, further down the river, had a great reputation for the excellence of its ships, but the young Merrick was more attracted by the countryside than by the bustling wharves. He seemed to feel he was dedicated to greater things than Hallowell had to offer.

At the age of fifteen he departed to find employment in Philadelphia with his maternal uncle, John Vaughan, who was carrying on a prosperous wine merchant's business at 109–111 Front Street. Vaughan, like his brother-in-law John Merrick, was a remarkable man who was to exercise a healthy influence upon his young protege. More than a mere substantial man of business, he was an important figure in Philadelphia. Among the offices he held were those of librarian and secretary of the American Philosophical Society.

Judged by our standards, Samuel Merrick was young to travel so far from home and to start upon an independent career, but in those days a boy assumed man's responsibilities at an early age. John Vaughan was a bachelor, and he gave some of the affection and interest of a parent to the young relative committed to his charge. Through him Merrick was brought into contact with some of the leading spirits of the day, like Stephen Girard, Richard Wistar, Isaac Norris, John Price Wetherill, and a host of lesser lights who passed through the home and offices of John Vaughan. In those days, Philadelphia was not only the largest city in the United States, it was also the principal seaport. From the charming rusticity of Maine, the boy was suddenly plunged into an atmosphere of wealth, refinement, and culture.

We need not conjecture upon Merrick's explorations of the nearby wharves along the riverside nor his country-boy's wonder over the incidents of the markets and the city streets. All we know about this period is that he applied himself conscientiously to his work in the counting house and earned his uncle's approbation. This much we know, for in 1820, when he was nineteen, Samuel Merrick was chosen by his uncle as partner in a new enterprise.

Uncle John was widely known for his kindness of heart and, while many worthy people benefitted by his generosity, there were others who took advantage of him. Some of these were business acquaintances who borrowed money from him as a last resort to support enterprises that were beyond any redemption. One of these concerns had been engaged in making fire engines. The business was inefficiently conducted and the time arrived when the proprietors could no longer meet their bills and were declared insolvent. John Vaughan then found himself unexpectedly in possession of an insolvent fire engine plant. The natural supposition would be that a man trained to mercantile life would shrink from engaging in an industry requiring a mechanical training. He might have sold the plant for what it would bring to someone better qualified. Instead, he chose to put his nephew in charge of it.

Why he should have taken this course we do not know, nor why he thought his nephew possessed the ability to salvage a derelict business. We can only surmise that some of the more far-seeing members of John Vaughan's circle had been discussing the Industrial Revolution that was in full blast in Europe, and had concluded that it must lead to the adoption of an industrial system in the United States if the country was to assume a place among the progressive nations. Vaughan had no practical knowledge of mechanics. Nor had Samuel Merrick, his partner.

It is even more perplexing to find the reason why the younger man accepted the offer. Having regard to the manners and customs of the time, to move from the counting house of a well established merchant into a factory was something of a social degradation. Perhaps the clue to the reason why he accepted the post dwelt in his character. Here was a challenge. Throughout his life Samuel Merrick never evaded a challenge. We shall see how he accepted one after another and, in a quiet efficient manner overcame the obstacles they presented. A sense of responsibility, a flexibility of mind, a close application to the matter in hand which distinguished all his undertakings, had probably become apparent during the four years Merrick worked with his uncle.

Accordingly he moved into the plant situated at the rear of St. James church and set about the task of rebuilding the derelict business. Under his management it prospered. The name of the company was changed to Merrick and Agnew when Merrick took John Agnew into partnership after his uncle's withdrawal. Fire engines under their name were widely sold because of the superior qualities of workmanship. An odd commentary upon the publicity methods of the time is reflected in a remark upon the merit of Merrick and Agnew's fire engines made in 1870. Instead of mentioning that these fire engines were coveted by the firemen and found a ready market, the speaker said that they were used by professors in New England colleges "as illustrations in mechanics and specimens of American ingenuity and workmanship."

There was, of course, a reason why these fire engines should be praised for this selection, for hitherto most American cities requiring fire fighting material had been compelled to purchase it from England. Having established a reputation in this field, Merrick retained an interest in fire engines long after he had ceased to make them. He took a personal and active part in the Philadelphia Fire Department, and it was under the pressure he exercised that steam fire engines were obtained for the city fire companies.

As soon as the business was set upon a sound financial basis, Merrick married Sarah Thomas. The marriage took place on Christmas Day, 1823, when the groom was twenty-two years old.

Merrick was acutely conscious of the need to improve the fire engines that had failed to keep his predecessors in business. He could not go on indefinitely offering the same models for sale, but if he had to effect improvements he had to learn a great deal about applied mechanics and hydraulics. Failing to find any institution where he might secure the necessary instruction he arrived at the conclusion that Philadelphia should have a place where young and old could obtain the instruction when the nation was in grievous need of mechanics. Being a man of action he set about promoting the establishment of just such an institution. The outcome, after two unsuccessful attempts, was the founding of The Franklin Institute. It is no disparagement of the support he received from his friends Professor Keating and Frederick Fraley to say that Samuel Merrick was the founder of the Institute. Evidently those who attended the meeting at which the Institute was founded were of this opinion, for he was nominated to the office of President. With characteristic modesty he declined the nomination and urged the election of a more experienced man. When, after serving for eight years, the first President resigned the office, Merrick succeeded him. He held the office for thirteen years, after which he thought the younger men should be given recognition.

He was not satisfied to have given an impulse to the movement, but was active in all phases of Institute work. He served on the investigating committees that were to furnish The Franklin Institute with an international reputation for scientific and technical research.

Meanwhile, he was taking an interest in other public affairs, and was not always to be found on the popular side. For example, the question of gas lighting was agitating the public mind. Baltimore, Boston, and New York had adopted it after installations in England had proved its efficiency, but there remained a powerful body of opinion firmly entrenched against its adoption in Philadelphia. The objections covered a wide field, ranging all the way from the danger of introducing a poisonous gas into the home to metaphysical protests against upsetting the natural order of day and night by converting night into day. The city of Richmond had made an effort at compromise by erecting a high tower on its Main Street in order to furnish the entire surrounding neighborhood with artificial light. A proposal was made to light Philadelphia in the same manner from a tower 300 feet high.

Samuel Merrick was an early convert to gas lighting. Realizing that the best way to secure its adoption was by direct action, he sought and obtained a seat on the Common Council. Here he was able to persuade his fellow councillors to abandon their endless discussion on the problem and to follow The Franklin Institute's example of appointing an investigating committee to inquire into all phases of the subject and to make recommendations for the council to follow. Merrick was made chairman of this committee. The procedure followed was exactly the same as that adopted by the Institute's committees of investigation. All the information that could be collected was assembled, the other cities where gas had been installed were consulted, and opinions were gathered from both sides of the controversy. When all this information was assembled, the committee analyzed the data and recommended the installation of a gas works and a system of distribution. Feeling that all the objections had been triumphantly overcome, the Council then invited Merrick to visit Europe for the purpose of consulting the experts who had a longer experience with the new form of lighting.

Merrick returned in 1834 and submitted a report (later issued in pamphlet form) presenting a comprehensive and lucid account of all he had seen and could learn from the European gas engineers. He outlined the requirements for the city and furnished an estimate of costs. Finally, he showed that an installation such as he recommended would be cheaper, cleaner, and more convenient than either candles or oil.

The exhaustive report was so convincing that his recommendations were adopted by the City Councils. His investigations in this country and his inquiries abroad had made him one of the best informed men upon the subject of illuminating gas and, as he had already proved his ability in management, he was appointed Chief Engineer of the city gas works. It is a tribute to his organizing ability and to his technical achievements that the gas works were built on the bank of the Schuylkill River, just above the Market Street bridge, and the lights turned on in 1836. It was remarked at the time that the operations had been performed without a single delay or fault.

In his report to the Councils in 1834, Merrick displayed a trait in his character that is reminiscent of Benjamin Franklin, that other New Englander who had made Philadelphia his home and had worked zealously for his adopted city and state. He wrote:

> I deem it an argument of no small moment in favor of this mode of lighting, that every material used in the fabrication of this gas will be a product of Pennsylvania labor. The bituminous coal from which it is to be made, may be drawn from the rich mines now open in the interior of the State; the fuel from the exhaustless body of anthracite; and the lime for purification from our own vicinity; and not a lamp will shed its rays over our streets which has not paid a tribute to the internal improvements of the State.

This observation not only shows his desire to develop the resources of the State but it proves his readiness to accept the responsibility for making innovations. The other towns that were making illuminating gas in this country were distilling it from whale oil and rosin. Merrick proposed to follow the English example of using coal, thereby releasing the gas works from reliance upon itinerant whalers. He was already in advance.

The trustees of the Gas Company were emphatic in the testimony they bore to the ability of their Chief Engineer when they came to present their annual report for 1836. In expressing their pride for the speed and efficiency with which the construction was carried out, they declared the operation to be so perfect and economical as to be "certainly unrivalled in the country." Exactly one year after the gas lights had been first turned on in the streets, Merrick asked to be relieved of his duties as they interfered with his private enterprises. He had already planned and had started extension of the gas works, and the trustees begged him to serve as their consultant. The stockholders voted that the sum of $600 be appropriated to provide him with silver plate as a mark of their esteem.

The gas works had not entirely distracted him from attention to other projects. Railroads were occupying the attention of many people at that time. Most locomotives and rails were being imported from England, but American engineers were of opinion that these rails were unnecessarily heavy. Merrick evidently shared this opinion, for he designed what he called a "trough rail" of perceptibly lighter weight. This was literally an inverted trough with flanges that would be bolted to the ties. The engineers protested he had gone from one extreme to the other and that his rail was too light for their purpose. Merrick knew his critics were in error in attributing a lack of strength to this type of rail. To convince them that his rail was strong he submitted it to the Committee on Science and the Arts of the Institute, whose members had designed and built the only available machine for testing the strength of metals. The Committee's report was entirely favorable, the members supporting Merrick's claims with reasoned arguments, but these failed to convince the critics and the trough rail did not find any acceptance.

No man with his energy, capacity, or range of interest could long remain satisfied to make fire engines. His natural ability was now supported by the confidence he had gained in the successful organization of the gas works, an enterprise that had been accomplished with a notable absence of waste, delay, or errors. He now yearned to exercise his talents in a broader field. The times were propitious. Iron was replacing wood in industries; railroads were spreading their iron web over the land; handcrafts were yielding place to machinery; water wheels were being replaced by steam engines; the steamboat was extending its range over the oceans. It was a time of inventive ferment.

Merrick resolved upon seizing the opportunities offered. Even while he was seemingly immersed in the gas works project he was planning the course he would follow as soon as he had completed this task. His fortunes had evidently taken an upturn, for he had bought a house facing on Northwest Penn Square. He had also taken as assistant a young man John Henry Towne, whose memory is kept alive in the Towne Scientific School of the University of Pennsylvania. Towne was then a youth of eighteen who had displayed a strong inclination toward the new profession of engineering. He developed some of Merrick's characteristics as an experimenter, builder, and innovator. Under his employer's supervision young Towne had aided on the drawings for the gas works, and by the time this work was completed he became Merrick's helper in laying the foundation for a new enterprise.

In the year 1836, as soon as he was liberated from the gas works project, Merrick opened the Southwark Foundry with Towne as his junior partner.

The Southwark Foundry was situated on the Pine Street Railroad (Washington Avenue) between 4th and 5th Streets in Philadelphia. At first it was intended only to do castings, but before it had been long established it became one of the city's most impressive industrial establishments, for it comprised a mechanics' shop of three floors, a smith's shop with 18 forges, a foundry, and a boiler shop. The foundry was not more extensive than most of its contemporaries, but Merrick saw to it the equipment was superior and that the workmanship was of the best. Some idea of the bold planning the partners had followed may be gleaned from the fact the firm was able to undertake the construction of the engines for the steam frigate *New Jersey*, for which they had to cast a bedplate weighing 14 tons, an achievement of the first magnitude for those days.

The most interesting unit of the new foundry was the boring machine. This was designed by Merrick and Towne with novel features that attracted a great deal of attention from engineers. It was capable of boring a cylinder 96 inches in diameter and 14 feet long. The efficiency and novelty of this boring machine gained for the inventors the award of the Scott Legacy Premium.

The products of the Southwark Foundry were many and varied. It made boilers, brass and iron castings, sugar mill machinery, furnaces, steam hammers, high and low pressure steam engines for land and marine service, and iron lighthouses for the U. S. Lighthouse Service. The recent celebration of Commodore Perry's voyage to Japan serves to remind us that his ship, the U. S. S. *Mississippi* was one of several vessels engined by this Foundry.

Another famous ship for which engines were built was the U. S. S. *Princeton*, the first ship of war to be fitted with a screw propeller. The engines had been designed by the famous engineer John Ericcson, who had the discrimination to appeal to Merrick when he needed intelligent co-operation and the finest workmanship. The 14-ft. bronze propeller and the new type of engines were beyond the resources of most other contemporary plants. The *Princeton* was the herald of a new type of marine engine, and the experience gained in building her placed Merrick and Towne in the most favored position to benefit from the demand for more speed in ocean-going vessels.

A notable achievement of the foundry in its later days was its contribution to the improvement in the design of stationary steam engines. The engines that had been built formerly had been badly proportioned and roughly made. They were slow running but made so sturdily that they would jog along indefinitely if fed with a little oil occasionally. Merrick and Towne were prominent in the effort to arrive at more compact models capable of operation at higher speeds. Further, they were pioneers of making interchangeable parts for steam engines.

During the first ten years of operation the foundry prospered but it was regretfully observed that the city was descending from its position of leadership owing to new avenues of communication being opened with the expanding West that were diverting trade to tidewater at New York, or to Baltimore over the thriving Baltimore and Ohio Railroad. The operation of a railroad by the State Works, by which it was hoped to attract traffic through Pennsylvania to the Atlantic Ocean, had met with failure and was incurring heavy losses. There were natural obstacles of imposing magnitude to be overcome in providing substitutes for the inclined planes that carried passengers and freight over the mountain barriers, and created the necessity for discharging and re-embarking them between railroads. Operations were hampered by the closure of the canals for three or four months in winter, and the periodical damage done by floods and freshets.

Merrick as a prominent member of the Board of Trade was keenly watching the situation, as were others. Together with the more discerning citizens he perceived the danger threatening when the Baltimore and Ohio Railroad applied to the legislature to complete its tracks to Pittsburgh. A powerful movement was set on foot to forestall the danger by building a railroad that would carry the traffic directly to Philadelphia. The outcome was the creation in 1846 of what is now the Pennsylvania Railroad. As soon as a Board of Directors was formed Samuel Merrick was elected President.

It was a foregone appointment. He was an outstanding figure in finance, for he had displayed exceptional ability in weathering the bank crisis that had almost paralyzed business in the city. His conduct of the Southwark Foundry had shown him to be possessed of exceptional qualities in the fields of management and engineering. He had furnished proof of exercising the sound judgment necessary to organize an extensive enterprise and the integrity to guide it along the path to success. He had met and accepted more than one challenge. Railroading was but another. Merrick tackled it with characteristic skill and energy. As president of the new railroad he was saddled with two burdensome tasks that overshadowed a multitude of detail. The first was to raise the necessary capital. His motive for assuming a position of grave responsibility was clearly stated in his first annual report.

> The absolute necessity of this road to the trade of Philadelphia is universally acknowledged. The completion of the Cincinnati and Sandusky road brings that city within three days ride of New York for eight months of the year. The trade of the Ohio River which once belonged to Philadelphia, is now diverted to New York by this new channel of the lakes. Hundreds of passengers daily pass over that road to New York. Where the travel goes, there goes trade.
>
> You are engaged in a great struggle for the trade of the West. To obtain it a portion of your earnings must be devoted to open the highway. Once open, it will maintain and enlarge itself. Railroads and Canals have built up New York . . . Boston has been built up by the same means, and if we expect to maintain our position, we must follow their example.

No one today would care to criticize this viewpoint nor claim there was a better alternative measure for rescuing the city and state from the strangulation of trade that was not only threatening but imminent.

It should be observed that Merrick, in forecasting expansion, was warning the prospective stockholders not to anticipate dividends, since all surplus funds were to be applied to the extension of the scheme without incurring debt. This principle of administering a business without burdening it with indebtedness was a practice he both advocated and practiced. But the public to whom his pleas for funds were addressed responded.

The second great task confronting him was the technical work of planning and surveying the route. The road had to be laid over the Allegheny Mountains, that natural barrier that had formerly been crossed by the time-consuming inclined planes.

Merrick remained in office for two years. During this time the capital had been raised from one million dollars to more than three and a half millions, and this when there was no return on the investment. Such was the confidence placed in the integrity and ability of Samuel Merrick as organizer and administrator. The future was to prove the confidence was not misplaced.

But he could no longer spare the time from the operation of his foundry and, having seen the railroad successfully launched he retired from the presidency, but remained on the Board of Directors. On the day after he had resigned from the principal office, Merrick had the satisfaction of witnessing the opening of the railroad for travel between Harrisburg and Lewistown. The way was almost open to the foot of the Alleghenies and plans were complete for the final achievement.

Although he must have felt he had terminated his railroading days, Merrick was recalled to them under peculiar circumstances. An effort was being made to construct a railroad from Sunbury to Erie, thus completing communication by rail between Philadelphia and the Great Lakes, through the great Pennsylvania coalfields. Although a number of prominent men were associated with this enterprise, it had encountered numerous failures and was on the verge of disaster. In their plight, the directors turned to Samuel Merrick as being the only man in their estimation who might retrieve the situation. In the year 1856 he was offered the position of president of this railroad. He confided to intimate friends his intention of declining the office, and this decision being conveyed to the directors and stockholders in confidence, the latter addressed to him a remarkable appeal. They assured him they were aware their request would present no attractions to him but,

> . . . There are occasions when purely personal motives ought properly to yield to public claims, and in the exigency which calls for an efficient and tried man to administer the affairs of this important work, we will address ourselves to you to assume the Presidency. Your perfect organization and successful administration of the affairs of the Pennsylvania Railroad Company are so well remembered by our fellow-citizens, that we are sure the whole community would hail with pleasure your acceptance of the proffered trust.

This blunt appeal to his public spirit could not be resisted by a man of Merrick's temperament. He yielded to the appeal and entered upon a task that was to prove the severest trial of his career. He threw himself into the work with his customary zeal and had no sooner devised plans for remodelling the railroad than the terrible commercial crisis of 1857 swept over the country. Private enterprises that had been long established were prostrated, new ones were wiped out in default of public or private aid. The Sunbury and Erie Railroad would have succumbed as so many other companies did had not Samuel Merrick sustained it by advances from his private funds. At the risk of ruining himself he saved the company.

This effort made severe inroads on his health and he was constrained to offer his resignation after serving the company during two anxious years, but he had the satisfaction of seeing it in a healthy condition and advancing toward completion.

During these years his responsibilities at the Southwark Foundry had doubled. He was alone there now, for John Towne had withdrawn from the partnership in 1847, and the proprietors were now listed as Merrick and Sons. The business had grown and the plant covered four acres of ground, with an additional ten acres of wharves on the Delaware waterfront. Merrick machinery was to be found everywhere on the American continent. There was much to occupy the head and the heart in his own enterprises but he was constantly engaged in public affairs. He was prominent on the Board of Trade, for whom he was delegated to speak at the celebration of the co-ordination of the city of Philadelphia in 1854; he was a Warden of the Port, a founder of the Western Savings Fund, and a member of several other institutions engaged in doing practical good for the city and its inhabitants.

That his interests were not confined to his immediate neighborhood was shown at the time of the Civil War. Profoundly conservative in all his principles he was deeply moved by the attitude of the South, and threw himself wholeheartedly into support of the Union cause. The Sanitary Commission engaged much of his attention, and he contributed generously to its funds which were applied to the care of the sick and wounded. He devoted so much of his time and energy to the Great Central Fair in aid of this object that his health was seriously impaired.

After the war, he gave his attention to the provision of schools in the stricken South, and took a lively interest in the education of the colored people. His contributions of money for the building of schools were on a munificent scale.

His benefactions will never be known to their full extent. Without resorting to any contrivances Merrick had an extraordinary gift for concealing his generosity. On learning from acquaintances of cases of individual distress he would give sums of money to his informant with the request that they be conveyed to the victims. On one occasion, when a friend was in need of money, he had no more than stated his predicament than Merrick wrote out a check for a thousand dollars. His benefactions were often calculated in the thousands, and became known to his family and most intimate friends only by accident.

Franklin and Merrick have many parallels in their lives. It is apparent from his choice of name for The Franklin Institute that the young Merrick was an admirer of the great Franklin, and during his lifetime he exhibited a desire to put into practice the doctrines of thrift, application to business, and benevolence to which his model had given expression. Both men after their deaths left a gap that could not easily be filled in the city of their adoption.

pH-COMPOSITION RELATIONS IN THE SYSTEM Na_2O-SiO_2-H_2O AT 20° C.*

BY

LESLIE R. BACON [1] AND JOHN H. WILLS [2]

ABSTRACT

New apparatus and technique for the accurate determination of electromotive force and pH data of solutions are described. These have been used in an extensive investigation at 20° C. of the pH-composition relationships of sodium hydroxide and sodium silicate solutions of the five $Na_2O:SiO_2$ molecular ratios 1:0.487, 1:1.009, 1:1.492, 1:1.996 and 1:3.015 (orthosilicate to trisilicate range). The results are of interest as a contribution toward understanding the constitution of silicate solutions, and wherever pH may be used for control purposes in the practical application of silicates.

At the trisilicate ratio where pH changes with time, pH is a function of the history of the silicate solution. Comparative colorimetric values by the Taylor indicators are given. Use of these indicators is hardly practicable at concentrations below 0.01 per cent Na_2O.

The density or specific gravity of sodium hydroxide and sodium silicate solutions varies with concentration ($A = \%\ Na_2O$) according to the relation

$$d = 1 + aA + bA^2 + cA^3$$

over a considerable range.

INTRODUCTION

This paper gives accurate data on the pH of the system Na_2O-SiO_2-H_2O covering the range from purely ionic, slightly viscous NaOH solutions to viscous, highly colloidal silicate solutions. These data aid in the understanding of the constitution of silicate solutions which find wide commercial uses. Data recorded in the literature on this system are neither as extensive nor as accurate as is desirable. Because there are reasons to believe that the method of preparation and subsequent history may influence the properties of silicate solutions, especially the more siliceous silicates, special attention was given to the details and refinement of pH measurements and to the history of the solutions.

I. EXPERIMENTAL METHODS

A. Apparatus

Three pieces of glass apparatus including new hydrogen gas saturators (Fig. 2) and calomel (Fig. 3) and hydrogen electrodes (Figs. 4 and 5) were especially designed so that the apparatus could be mounted in a water thermostat and cleaned without removal from the water thermostat. General assembly is shown in Fig. 1.

---

* This work was done at the Philadelphia Quartz Company by Dr. Bacon and completed in 1939. The original report has been edited for publication by Dr. Wills.

[1] Wyandotte Chemicals Corporation, Wyandotte, Mich.; formerly with the Philadelphia Quartz Company, Philadelphia, Pa.

[2] Philadelphia Quartz Company, Philadelphia, Pa.

1. Gas Saturators

In the gas saturation, hydrogen, introduced through the central tube and tip, spirals upward around the central baffle and exerts a pumping action on the liquid. When not in operation, liquid fills the vessel to the central portion of the spiral (50 ml.). In service the bubbling action raises the level of the liquid to overflow through the by-pass, providing continuous recirculation. The vessel is pre-rinsed and filled through the capillary tube with aliquots of the solution under pH measurement, and emptied by suction. Protection against change of concentration was thus obtained by presaturating the hydrogen gas at the vapor tension existing in the hydrogen electrode vessel.

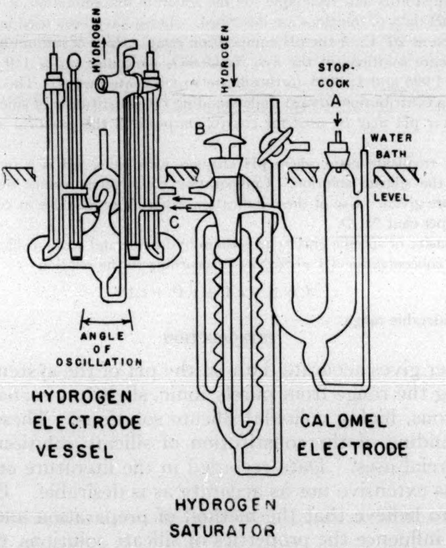


FIG. 1. Working relations of the hydrogen and calomel electrodes.

Another saturator filled with KOH-$KMnO_4$ solution was used for removing any acidic or reducing gases from the hydrogen supply. These bubblers functioned over long periods with satisfaction and little attention.

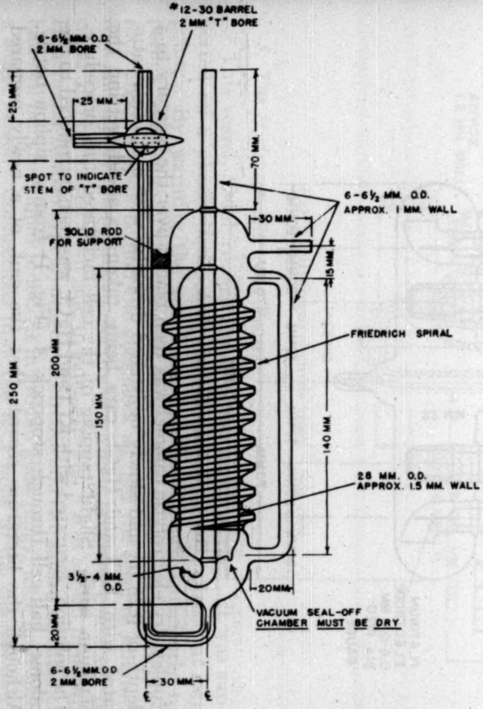

#12-30 BARREL
2 MM. "T" BORE
6-6½ MM. O.D.
2 MM. BORE
25 MM.
25 MM.
SPOT TO INDICATE
STEM OF "T" BORE
70 MM.
6-6½ MM. O.D.
APPROX. 1 MM. WALL
30 MM.
SOLID ROD
FOR SUPPORT
15 MM.
FRIEDRICH SPIRAL
250 MM
200 MM
150 MM
140 MM
28 MM. O.D.
APPROX. 1.5 MM. WALL
3½-4 MM.
O.D.
20 MM
20 MM
VACUUM SEAL-OFF
CHAMBER MUST BE DRY
6-6½ MM. O.D
2 MM. BORE
30 MM.
℄
℄

Fig. 2. Hydrogen gas presaturator.

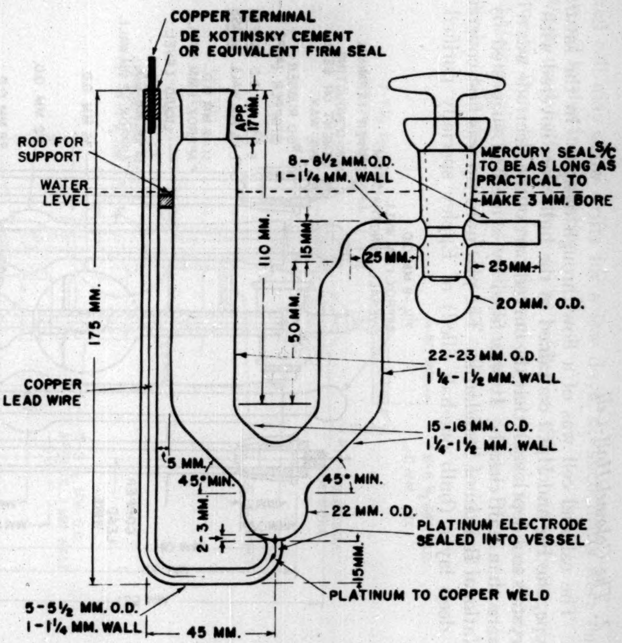

COPPER TERMINAL
DE KOTINSKY CEMENT
OR EQUIVALENT FIRM SEAL
APP. 17 MM.
ROD FOR
SUPPORT
WATER
LEVEL
8-8½ MM. O.D.
1-1¼ MM. WALL
MERCURY SEAL S/C
TO BE AS LONG AS
PRACTICAL TO
MAKE 3 MM. BORE
110 MM.
15 MM
25 MM.
25 MM.
20 MM. O.D.
175 MM.
50 MM.
22-23 MM. O.D.
1¼-1½ MM. WALL
COPPER
LEAD WIRE
15-16 MM. O.D.
1¼-1½ MM. WALL
5 MM.
45° MIN.
45° MIN.
22 MM. O.D.
2-3 MM.
PLATINUM ELECTRODE
SEALED INTO VESSEL
15 MM.
PLATINUM TO COPPER WELD
5-5½ MM. O.D.
1-1¼ MM. WALL
45 MM.

Fig. 3. Calomel electrode.

2. *The Calomel Half-Cell*

The calomel cell was of a flow-through design. In the saturated type, the Hg and HgCl contained in the bulb were covered with KCl crystals to a depth such that the curved part of the electrode was a little more than half closed. It was filled by a method suggested by the National Bureau of Standards. The platinum tip was first covered by a deep layer (bulb one-half filled) of Eppley specially purified Hg. Eppley dry electrolytic calomel was then placed over the mercury to a depth of 0.25 in. and slowly moistened dropwise with a solution saturated with KCl and HgCl, care being taken to avoid wetting the platinum wire. Slight tapping of the cell facilitated this procedure. The solution, saturated with KCl and HgCl, was then introduced to the calomel half-cell through stopcock *A* (Fig. 1) from a siphon bottle. Although the literature records that the emf. of a newly prepared

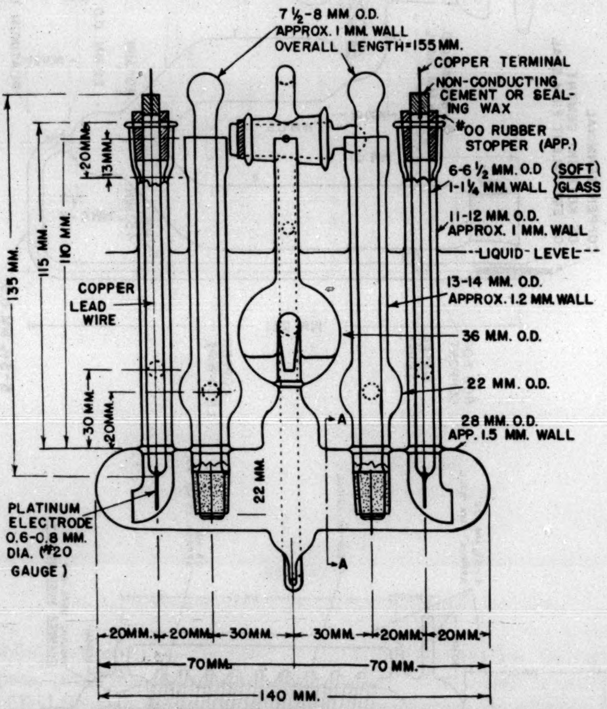

7 ½-8 MM O.D.
APPROX. 1 MM. WALL
OVERALL LENGTH=155 MM.
COPPER TERMINAL
NON-CONDUCTING CEMENT OR SEALING WAX
#00 RUBBER STOPPER (APP.)
6-6 ½ MM. O.D
1-1¼ MM. WALL
(SOFT GLASS)
11-12 MM. O.D.
APPROX. 1 MM. WALL
LIQUID LEVEL
13-14 MM. O.D.
APPROX. 1.2 MM. WALL
36 MM. O.D.
22 MM. O.D.
28 MM. O.D.
APP. 1.5 MM. WALL
20 MM.
13 MM.
135 MM.
115 MM.
110 MM.
COPPER LEAD WIRE
30 MM.
20 MM.
22 MM.
A
A
PLATINUM ELECTRODE 0.6-0.8 MM. DIA. (#20 GAUGE)
20 MM.
20 MM.
30 MM.
30 MM.
20 MM.
20 MM.
70 MM.
70 MM.
140 MM.

FIG. 4. Hydrogen electrode vessel. Front view.

calomel cell may show drifts for a few days, no such drifts were encountered in this investigation.

When used half-cells were to be refilled it was found necessary to remove Hg from the Pt contact by anodic electrolysis in dilute HNO_3. The black coating which sometimes appears on the Pt can thus be removed. It is also essential that the cell be clean and baked out.

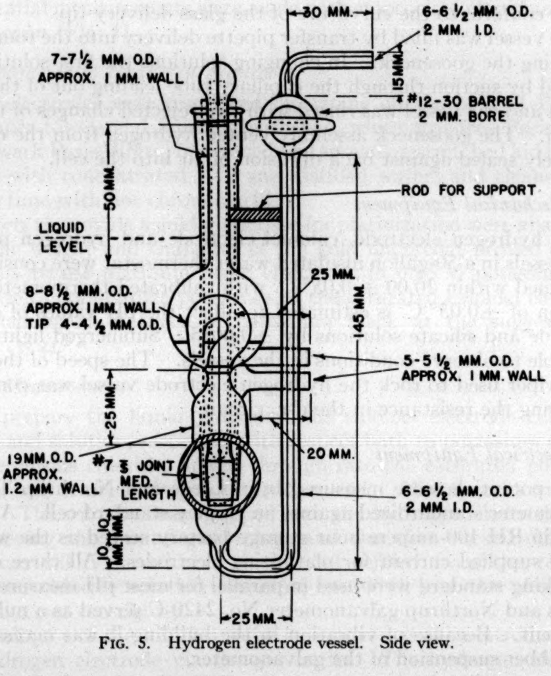

30 MM.
6-6 ½ MM. O.D.
2 MM. I.D.
15 MM.
7-7 ½ MM. O.D.
APPROX. 1 MM. WALL
#12-30 BARREL
2 MM. BORE
50 MM.
ROD FOR SUPPORT
LIQUID LEVEL
25 MM.
8-8 ½ MM. O.D.
APPROX. 1 MM. WALL
TIP 4-4 ½ MM. O.D.
145 MM.
5-5 ½ MM. O.D.
APPROX. 1 MM. WALL
25 MM.
19 MM. O.D.
APPROX.
1.2 MM. WALL
#7 ₮ JOINT
MED.
LENGTH
20 MM.
6-6 ½ MM. O.D.
2 MM. I.D.
10 MM.
10 MM.
25 MM.

FIG. 5. Hydrogen electrode vessel. Side view.

3. *The Hydrogen Electrode Vessel*

This vessel, 50-ml. capacity, was of the rocking-type. It was supported on a plastic backboard with blocks of paraffin placed at key points. By warming the vessel the paraffin softened sufficiently to permit positioning of the vessel in place. The vessel was mounted to rock about a pivot behind the gooseneck so that the level of solution rose and fell on the hydrogen electrodes without breaking contact. It could also be rocked from side to side. In Fig. 4 it is shown how two Hildebrand-Bunker type hydrogen electrodes and two saturated calomel half-cell bridges (liquid junctions in ground glass plugs) were built into a single vessel. Connections from the KCl wells to the calomel half-cells were made through the side-arms (Figs. 1, 4, 5). The plugs were not fitted so tightly as to cause excessive resistance at the junction but sufficiently to prevent leakage. The rocking of the vessel and the bubbling of hydrogen caused an alternating exposure of the hydrogen electrodes to solution and hydrogen gas, a most desirable condition for satisfactory operation. It was necessary to maintain liquid contact during emf. measurements and desirable to use long Pt electrode tips bent to conform to the curvature of the glass delivery tip.

The vessel was filled by transfer pipette delivery into the round bulb containing the gooseneck. In changing solutions the first solution was removed by suction through the capillary tube leading out of the sump (Fig. 5) and the vessel was rinsed with two rejected changes of the new solution. The gooseneck assembly vented hydrogen from the cell and effectively sealed against back diffusion of air into the cell.

4. *Mechanical Equipment*

The hydrogen electrode, calomel electrode and hydrogen presaturator vessels in a 50-gallon insulated water thermostat were consistently maintained within 20.00 ± 0.05° C. with calibrated thermometers. A variation of ±0.05° C. is estimated to affect the pH values of sodium hydroxide and silicate solutions by ±0.0015. Submerged lights made it possible to observe conditions in the vessels. The speed of the windshield wiper used to rock the hydrogen electrode vessel was controlled by varying the resistance in the d-c. circuit.

5. *Electrical Equipment*

Cell potentials were measured by a Leeds and Northrup Type K potentiometer standardized against an Eppley standard cell. A Philco Type 136 RH 100-ampere-hour storage battery served as the working cell and supplied current for platinizing electrodes. All three cells of the working standard were used in parallel for most pH measurements. A Leeds and Northrup galvanometer No. 2420-C served as a null point instrument. Because of vibration in the building it was necessary to use a rubber suspension of the galvanometer.

6. *Arrangement of Equipment*

Cylinder hydrogen was washed by passing through a saturator, then passed over heated platinized asbestos for the removal of oxygen and through a header to the saturators immersed in the thermostat for equalization of the temperature and water vapor pressure with that of the solution in the hydrogen electrode cell. Pure gum rubber tubing was used to make all connections between vessels.

The working assembly required two saturators and two saturated calomel electrodes. Between the calomel electrodes and the wells for the ground plugs, T stopcocks with short arms (not shown in Fig. 1) were interposed at *C*. The side arms of these stopcocks were interconnected so that potentials of the two saturated calomel electrodes occasionally could be checked directly against each other without involving the junctions at the ground glass joints. The potentials between the calomel cells were checked through the hydrogen electrode vessel for each test solution whose emf. was measured. The ground glass joints were loosened and flushed whenever solutions were changed. All potential measurements were made with stopcock *A* closed and stopcock *B* open (Fig. 1).

B. Preparation of Hydrogen Electrodes

Old electrodes were reserviced by wiping off the bulk of Pt black with paper towels and finishing with fine, white, grit-free Paris White (Southwark Paris White) moistened and spread on stretched kid leather; washed with concentrated HCl and distilled water; and cleaned from time to time with hot chromic acid.

Efforts to provide a gold plate base for platinization were abandoned when the weight and character of the gold undercoat proved critical. Attempts to use platinum black electrodes in 0.05 M potassium acid phthalate to standardize potentials of the saturated calomel electrodes were abandoned in favor of palladium black at the suggestion of Hamer (**1**, **2**).[3]

C. pH Measurement

To prepare the liquid junctions, the calomel electrode cells were opened and solution, saturated with respect both to potassium chloride and mercurous chloride, flowed through into the saturated potassium chloride well of the hydrogen electrode vessel. All stopcocks were then closed. The solution was dropped into the hydrogen electrode vessel by loosening the ground glass plugs and was then sucked out. The potassium chloride well was rinsed and refilled from the top with potassium chloride solution saturated at 20° C. and the plug seated firmly but not too tightly.

An aliquot of the solution to be measured was then introduced into the hydrogen electrode via the gas outlet. The hydrogen electrodes were usually introduced at this point, so that the first filling served to wash out the cell and effect neutralization of any acid residual in the platinum mass. The cell was again carefully rinsed and refilled, usually this time through the capillary suction tube.

Hydrogen flow and rocking were started and continued to the attainment of equilibrium, usually fifteen minutes. With some very dilute solutions the period was found to be two hours.

Differential measurements between the calomel cells C_1 and C_2 were made against each other directly and also through the hydrogen electrode vessel during this period.

---

[3] The boldface numbers in parentheses refer to the references appended to this paper.

After securing stable agreement well within 0.0006 v (less than 0.01 unit in pH effect), the four combinations of hydrogen and calomel (HC) cells were measured in a definite order. For the detailed representation of these cells see Table I.

TABLE I.—*Electromotive Force Cells.*

Cell Designation*	Plug No. 1	Plug No. 2	
HC	In	In	pH measurements
C_1C_2	In	In	Through H_2 electrode vessel filled with saturated KCl
C_1C_2 (1)	In	Out	
C_1C_2 (2)	Out	In	
C_1C_2 (3)	Out	Out	

* Where HC indicates cells of type:

Pt, H_2 (1 atm) | Unknown Soln. ‖ KCl (s) | KCl (s), HgCl | Hg

C_1 or C_2 represents Hg | HgCl, KCl (s) | KCl (s) ‖.

Voltages in most cases could be recorded significantly to the fifth decimal place (nearest hundredth millivolt), and the E values for the four cells H_1C_1, H_2C_2, H_2C_1 and H_1C_2 were averaged after measurement and were rounded off to the nearest tenth millivolt for pH calculation. At 20° C., 0.1 mv. = 0.0017 pH or 0.01 pH = 0.0006 v. (nearly).

D. pH Standardization

pH values were calculated from the four cell average electromotive force measurements by

$$\mathrm{pH} = \frac{E + E_{\mathrm{bar}} - E_{\mathrm{cal}}}{0.00019832T} \tag{1}$$

where E is the observed potential, E_{bar} is the potential correction to be applied to readjust to the basis of 760 mm. hydrogen gas pressure, E_{cal} is the potential of the reference calomel cell and T is the absolute temperature.

Barometric corrections were calculated from the formula

$$E_{\mathrm{bar}} = \frac{0.0001983T}{2} \log \frac{760}{X} = 0.0291\ (2.8808 - \log X) \tag{2}$$

where X is the corrected barometric pressure in millimeters less the vapor pressure of water at 20° C. Vapor pressure lowering by the solute is of unknown value but small, and was neglected.

Our results have been computed on the basis of the Leeds and Northrup formula (3):

$$\mathrm{pH} = \frac{E + E_{\mathrm{bar}} - 0.2648 + 0.00076t}{0.0001983T} \tag{3}$$

where t is centigrade temperature and $T = 273.1 + t$. At 20° C. this becomes

$$\mathrm{pH} = \frac{E + E_{\mathrm{bar}} - 0.2496}{0.0581}. \tag{4}$$

The Leeds and Northrup value, $E_{\mathrm{cal}} = 0.2496$ forms a suitable working basis until authorities are agreed on primary (4) standards. Recalculation may then become warranted.

If, for the 0.05 M potassium acid phthalate cell, a pH of 4.01 (5) for standardization of the saturated calomel electrode is accepted, the corresponding value for E_{cal} is 0.2472. Use of this value would increase the pH values calculated in this paper by about 0.04 unit. Actual standardization trials were unsuccessful because of a continuous drift ascribed to catalytic reduction.

Differential drift between the two saturated reference electrodes was checked with a cell set up with no flow of KCl solution through the cell. Potential differences always proved negligible in terms of pH measurements.

E. Operating Experiences

The effects on emf. of the C_1C_2 arrangement with alkaline solutions in the cell were frequently greater than when filled with saturated KCl, but were normally well within the 0.6-mv. limits desired for pH measurements. The effect of two ground glass plugs in the hydrogen electrode vessel, the normal operating condition, was found to be not the simple summation of the effects of each plug taken separately, but less. The introduction of a high resistance into the circuit, as by closing the side arm stopcock of the saturated calomel cell, threw the potential off balance with the galvanometer used.

F. Precision and Accuracy of Results

Any erratic behavior in the independent measurement of emf. from four HC cell combinations forewarned of experimental difficulties usually subject to correction. Averaging the four emf. measurements further minimized or eliminated errors by cancellation. The following relations

1. $H_1H_2 = H_1C_1 - H_2C_1$
2. $H_1H_2 = H_1C_2 - H_2C_2$
3. $C_1C_2 = H_1C_2 - H_1C_1$
4. $C_1C_2 = H_2C_2 - H_2C_1$

are fundamental and may be solved for eight derived HC relations. When added together and averaged, the result is $(H_1C_1 + H_2C_1 + H_1C_2 + H_2C_2)/4$. This results in the cancellation from the final result of any difference in the potentials of the two hydrogen electrodes or the two calomel electrodes, along with any steady junction potentials arising at the ground glass plugs, all of which are uncertainties in the measurement of any single HC cell. The averaged results should be as dependable as the approach to true equilibrium, assigned values of the calomel electrode potential and instrumentation errors permit.

In tables where pH is reported to the nearest 0.01 pH unit, only a sixth to a sixtieth of the precision is retained with which electromotive force measurements were made. In aiming at a precision of 0.01 pH unit it is believed that values good to ±0.05 pH unit or better (metasilicate an exception) have been attained for concentrations up to 0.001 per cent Na_2O, and to ±0.02 unit or better for higher concentrations of sodium hydroxide and silicates up to the ratio $Na_2O:SiO_2 = 1:1.5$.

The disilicate and more siliceous ratios change pH with time and the earliest measurable pH values after dilution (30–40 min.) for the trisilicate, are used as a basis for interpolation and extrapolation. The 1:1.8 ratio and three others between the disilicate and trisilicate ratios ought to be measured as well as silicates of different history.

II. EXPERIMENTAL METHODS

G. Materials. Preparation of Solutions

1. Water

Conductivity water, prepared by accepted methods, was required for reliable results at all dilutions. Very sensitive colorimetric tests (6) proved that solutions of silica occurred if the bottles of water were kept too long.

2. NaOH Stock Solution

Solutions of sodium hydroxide needed in this investigation were prepared by electrolysis through mercury amalgam from analytical reagent NaOH in an electrolytic cell in which the amalgam was formed and decomposed into NaOH solution completely isolated from the atmosphere and in contact only with pure Ni electrodes, pure gum tubing and paraffin. A sufficient stock for the entire program analyzed 13.503 per cent Na_2O by titration with standardized HCl using methyl orange as indicator.

A test for CO_2 was made by flowing the NaOH solution beneath a layer of melted paraffin into a clear, filtered barium hydroxide solution. Thus protected from exposure to air, no turbidity developed and it is estimated that the CO_2 concentration did not exceed 0.001 per cent. All alkaline solutions were protected at all times from contact with glass by silvering or paraffin coatings to insure that the SiO_2 and Na_2O contents would remain unchanged.

3. *Sodium Silicate Stock Solutions*

pH measurements were made on silicate solutions closely approximating the following ratios: orthosilicate, metasilicate, 1 Na_2O:1.5 SiO_2 (molecular ratio), disilicate and trisilicate.

Analyses of the sodium metasilicate showed the following averages:

	Per Cent
Na_2O	28.825
SiO_2	28.193
H_2O	42.490
Al_2O_3	0.010
Fe_2O_3	0.003
TiO_2	0.003
CaO	0.001
MgO	0.016
CO_2	0.133
Na_2O (in Na_2CO_3)	0.187
Cl	0.020
SO_3	0.026
NaCl	0.031
Na_2SO_4	0.046
	99.938

Molecular ratio Na_2O: SiO_2 : H_2O
0.991:1.000:5.028

A subsequent test at the time of usage showed 0.145 per cent CO_2 which was not regarded as significant. The stock solution prepared contained 7.653 per cent Na_2O. For the orthosilicate stock solution, a 50 per cent solution of Merck's reagent sodium hydroxide was rapidly filtered by suction through purified asbestos, protected from the atmosphere. The CO_2 content of such a solution should not exceed 0.07–0.15 per cent Na_2O as carbonate according to Han (7) and many other references consulted. Analysis showed 37.85 per cent Na_2O. 346 g. of this solution and 1543 g. of the metasilicate crystals were combined.

The stock solutions of more siliceous ratios were all made up at room temperature from the above sodium hydroxide solution and "E" sodium silicate of the following analysis:

	Per Cent
Na_2O	8.672
SiO_2	28.02
H_2O	63.07
Al_2O_3	0.107
Fe_2O_3	0.014
TiO_2	0.009
CaO	0.010
MgO	0.024

(*Continued on next page.*)

CO_2 (as Na_2CO_3)	0.017
SO_3 (as Na_2SO_4)	0.029
Cl (as NaCl)	0.067
	100.039
Mol Ratio $Na_2O:SiO_2$	1:3.334
Viscosity	34 seconds
Baumé	39.95°
Clearness	36 + cm

The containers were cooled to remove the heat of mixing and prevent softening of the paraffin linings. The Na_2O concentration of each stock solution, held for a month before use, is the highest value appearing in the respective Tables II–VII.[4]

4. *Dilution Technique*

These stocks were diluted quantitatively to form "head" solutions at nearly equal logarithmic intervals. Lower concentrations were fixed by dilution of the head solutions after periods up to two months at ten-fold volumetric intervals with conductivity water.

Fifty milliliters of the "head" solutions, at 20° C., were pipetted into a 500-ml. volumetric flask and brought to the mark. Emf. measurements were made and dilution repeated by ten-fold steps to concentrations as low as practicable. This practice was convenient, practical, and helped to detect and prevent accumulation of certain kinds of systematic errors.

The 500-ml. volumetric flasks were silvered for protection of the alkaline solutions against contamination by silica. Windows were scraped in opposite sides of the neck at the 500-ml. mark for calibration and use. A Lowy automatic pipette of 50-ml. capacity was calibrated but not silvered as time of exposure to the solutions was insignificant.

Na_2O concentrations below 1 per cent were calculated from the known dilution ratios and specific gravities of the original solutions by the methods of section H.

H. Specific Gravity—Composition Relations

Since titration was impracticable for solutions containing less than approximately 1 per cent Na_2O, the specific gravity-concentration data of Table VIII were first obtained. For the silicate of 1:3.015 molecular $Na_2O:SiO_2$ ratio the data were interpolated from highly precise charts of the Philadelphia Quartz Company; the other data are new. Specific

---

[4] To save space, about half the points determined have been retained in the tables. If question arises, the Philadelphia Quartz Company can supply the intermediate data. We believe that the data included will be sufficient for most purposes.

TABLE II.—*Sodium Hydroxide: pH-Concentration Data at 20.00 ± 0.05° C.*

Volume Normality	% Na_2O	$E + E_{bar}$	pH Electrometric
5.188	13.503	1.1069	14.76
2.065	5.913	1.0740	14.19
1.1256	3.336	1.0590	13.93
0.5361	1.627	1.0422	13.64
0.2098	0.6456	1.0211	13.28
0.04666	0.1446	0.9872	12.69
0.02097	0.06504	0.9681	12.37
0.01307	0.04055	0.9575	12.19
0.003482	0.01081	0.9249	11.625
0.001306	0.004054	0.9007	11.21
0.000465	0.001444	0.8698	10.67
0.000209	0.000650	0.8473	10.29
0.0000464	0.000144	0.7899	9.30
0.0000209	0.0000650	0.74	8.44
0.0000046	0.0000144	0.6909	7.60

Density (20° C./4° C.): Basic data from I.C.T.
Volume Normality: $N_v = \%\ Na_2O \times 1000 \times \text{Density}/30.997$.
Weight Normality: $N_w = 32.263\%\ Na_2O/(1 - 1.2907\%\ Na_2O)$.

TABLE III.—*Sodium Silicate: pH-Concentration Data at 20.00 ± 0.05°* C.

Volume Normality	% Na_2O	$E + E_{bar}$	pH Electrometric
	$Na_2O:SiO_2 = 1:0.487$ (Mol. Ratio) $= 1:0.472$ (Wt. Ratio)		
—	13.75	1.0948	14.54
	$Na_2O:SiO_2 = 1:0.507$ (Mol. Ratio) $= 1:0.491$ (Wt. Ratio)		
2.747	7.488	1.0694	14.10
1.0132	2.984	1.0443	13.67
0.3936	1.196	1.0244	13.33
0.1652	0.5084	1.0062	13.02
0.06185	0.1914	0.9857	12.66
0.01649	0.05115	0.9557	12.15
0.006173	0.01916	0.9317	11.73
0.001645	0.005109	0.8999	11.19
0.000616	0.001913	0.8727	10.72
0.000272	0.0008457	0.8524	10.37
0.000164	0.0005098	0.8391	10.14
0.000061	0.0001909	0.8030	9.52
0.000039	0.0001213	0.7824	9.17
0.000027	0.0000894	0.7602 ± 0.0015	8.78 ± 0.03
0.000016	0.0000509	0.7095 ± 0.0050	7.91 ± 0.09

Sp. Gr. (up to 4% Na_2O) = 1 + 0.0183% Na_2O.
$N_v = 32.203$ (Sp. Gr. 20° C./20° C.) (% Na_2O).

$$N_w = \frac{1000 \times 1.491\%\ Na_2O}{46.222(1 - 1.491\%\ Na_2O)}.$$

TABLE IV.—*Sodium Silicate: pH-Concentration Data at 20.00 ± 0.04° C.*

Volume Normality	% Na_2O	$E + E_{bar}$	pH Electrometric
	$Na_2O:SiO_2$ = 1:1.009 (Mol. Ratio) = 1:0.978 (Wt. Ratio)		
2.893	7.653	1.0548	13.85
0.832	2.450	1.0268	13.37
0.3138	0.954	1.0071	13.03
0.1237	0.3810	0.9894	12.73
0.03337	0.1034	0.9627	12.27
0.01964	0.0609	0.9485	12.02
0.00765	0.02373	0.9276	11.66
0.00334	0.01037	0.9088	11.34
0.001334	0.004141	0.8861	10.95
0.000497	0.001544	0.8631	10.55
0.000233	0.000724	0.8439	10.22

The following attempts to measure $E + E_{bar}$ at very low concentrations are not dependable but fix an order of values (see Fig. 8).

Volume Normality	% Na_2O	$E + E_{bar}$	pH Electrometric
0.000197	0.000612	0.8350	10.06
	0.000238	0.8136	9.70
	0.0001063	0.7706	8.96
0.000020	0.0000613	—	8.3–8.6

Sp. Gr. (up to 6% Na_2O) = 1 + 0.02243% Na_2O.
N_v = 32.203 (Sp. Gr. 20°/20° C.) (% Na_2O).

$$N_w = \frac{1000}{61.298} \times \frac{1.978\% \ Na_2O}{(1 - 1.978\% \ Na_2O)}.$$

TABLE V.—*Sodium Silicate: pH-Concentration Data at 20.00 ± 0.05° C.*

Volume Normality	% Na_2O	$E + E_{bar}$	pH Electrometric
	$Na_2O:SiO_2$ = 1:1.492 (Mol. Ratio) = 1:1.446 (Wt. Ratio)		
5.959	13.327	1.0435	13.66
1.858	5.101	1.0255	13.35
0.7164	2.114	1.0072	13.03
0.3237	0.9815	0.9913	12.76
0.1003	0.3092	0.9684	12.37
0.04569	0.1414	0.9517	12.08
0.01850	0.05737	0.9326	11.75
0.006850	0.02126	0.9104	11.37
0.003221	0.009999	0.8933	11.07
0.000999	0.003101	0.8668	10.62
0.000455	0.001412	0.8494	10.32
0.000184	0.0005720	0.8326	10.03
0.000100	0.0003095	0.8146	9.72
0.000045	0.0001409	0.7836	9.19

Sp. Gr. (up to 13% Na_2O based on new data).

$$d\ (20°/20°\ C.) = 1 + 0.02425A + 0.000235A^2 + 0.00001A^3.$$

N_v = 32.203 (Sp. Gr. 20°/20° C.) (% Na_2O).

$$N_w = \frac{1000 \times 2.446\% \ Na_2O}{75.802\ (1 - 2.446\% \ Na_2O)}.$$

TABLE VI.—*Sodium Silicate: pH-Concentration Data at 20.00 ± 0.05° C.*

Volume Normality	% Na_2O	$E + E_{bar}$	pH Electrometric
	$Na_2O:SiO_2$ = 1:1.996 (Mol. Ratio) = 1:1.934 (Wt. Ratio)		
5.376	12.535	0.9931	12.79
1.846	4.993	0.9849	12.65
0.4597	1.373	0.9664	12.33
0.1842	0.5630	0.9511	12.07
0.06673	0.2060	0.9292	11.69
0.03179	0.09843	0.9120	11.40
0.01838	0.05699	0.9011	11.21
0.003171	0.009846	0.8728	10.72
0.0006634	0.002060	0.8485	10.30
0.0001830	0.0005683	0.8246 ± 0.0012	9.89 ± 0.02
0.0000662	0.0002056	0.8016 ± 0.0030	9.50 ± 0.05
0.0000316	0.0000981	0.7647[a] ± 0.005	8.86 ± 0.09

[a] Validity open to experimental question. Point falls well on general curve, however.

Sp. Gr. (up to 12% Na_2O based on Alkali-Silica-Ratio Chart).

Sp. Gr. 20°/20° C. = $1 + 0.0288A + 0.000036A^2$.

$N_v = 32.203$ (Sp. Gr. 20° C./20° C.) (% Na_2O).

$$N_w = \frac{1000 \times 2.934\%\ Na_2O}{90.937\ (1 - 2.934\%\ Na_2O)}.$$

TABLE VII.—*Sodium Silicate: pH-Concentration Data at 20.00 ± 0.05° C.*

Age Head Solution	Intermediate Dilution	Volume Normality	% Na_2O	$E + E_{bar}$	pH Electrometric
		$Na_2O:SiO_2$ = 1:3.015 (Mol. Ratio) = 1:2.921 (Wt. Ratio)			
24 da.	0 da.	4.211	9.370	0.9162	11.47
28	0	2.317	5.879	0.9189	11.51
33	0	1.335	3.660	0.9190	11.52
28	0	0.2312	0.7010	0.9083	11.33
60	0	0.08283	0.2550	0.8945	11.09
24	0	0.04192	0.1296	0.8794	10.83
33	3	0.01329	0.04121	0.8640	10.57
59	3 hr.	0.005113	0.01587	0.8422	10.19
28	1 da.	0.002302	0.007147	0.8429	10.21
60	4 hr.	0.000825	0.002561	0.8329	10.03
33	3 da.	0.000132	0.000411	0.8148	9.72
24	1	0.000042	0.000129	0.7820	9.16

d = Sp. Gr. 20° C/20° C. = $1 + 0.0342A + 0.00034A^2 + 0.000055A^3$.

$N_v = 32.203$ (Sp. Gr. 20°/20° C.) (% Na_2O).

$$N_w = \frac{1000 \times 3.921\%\ Na_2O}{121.538\ (1 - 3.921\%\ Na_2O)}.$$

gravity data (20°/20° C.) for sodium hydroxide were calculated from ICT (8).

Specific gravity or density relations are represented as a function of

TABLE VIII.—*Density—Composition Data for Silicates.*

$\%Na_2O$ A	Sp. Gr. 20° C/20° C.	$\%Na_2O$ A	Sp. Gr. 20° C/20° C.
$Na_2O:SiO_2$ = 1:0.507 (Mol. Ratio)		$Na_2O:SiO_2$ = 1:1.996 (Mol. Ratio)	
0	1.0000	0.48	1.0140
1.196	1.0219	1.49	1.0432
1.861	1.0339	2.56	1.0741
2.984	1.0544	3.96	1.1154
4.729	1.0866	5.37	1.1600
7.488	1.1390	6.81	1.2083
		8.38	1.2609
$Na_2O:SiO_2$ = 1:1.492 (Mol. Ratio)		$Na_2O:SiO_2$ = 1:3.015 (Mol. Ratio)	
0	1.0000	0.82	1.0284
1.375	1.0340	1.66	1.0584
2.114	1.0523	2.56	1.0902
2.909	1.0723	3.46	1.1240
5.101	1.1308	4.38	1.1600
6.931	1.1823	5.28	1.1983
8.245	1.2212	6.22	1.2393
10.541	1.2939	7.15	1.2832
13.327	1.3886	8.13	1.3303
		9.11	1.3810
		10.13	1.4356

Na_2O over fairly wide concentration ranges with considerable accuracy by equations of the form

$$d = 1 + aA + bA^2 + cA^3 \tag{5}$$

$$\rho = 0.9982 + a'A + b'A^2 + c'A^3 \text{ (at 20° C.)} \tag{6}$$

where d refers to specific gravity, ρ to density and A to per cent Na_2O, and a, b, c and a', b', c' are constants for a given $Na_2O:SiO_2$ ratio and temperature. Table IX shows the values determined for these con-

TABLE IX.—*Constants for Density and Specific Gravity Equation.*

Specific Gravity, 20° C./20° C.; $d = 1 + aA + bA^2 + cA^3$

Molecular Ratio $Na_2O:SiO_2$	Constants a	b	c	A (Na_2O) Upper Limit %	Max.* Error in d
$\propto$ (NaOH)	0.01424	0.0000	0.0000	11	0.0005
1:0.507	0.0183	0.0000	0.0000	4	0.0002
1:0.996	0.02443	0.0000	0.0000	6	?
1:1.492	0.02425	0.000235	0.00001	13	0.0005
1:1.996	0.0288	0.0000	0.000036	7	0.0009
1:1.3015	0.0342	0.00034	0.000055	9.7	0.0012

* Maximum difference between experimental and calculated values.

stants, the upper Na_2O limits and the maximum differences between experimental and calculated values over the applicable ranges. a and a', b and b' and c and c' have the same values under identical conditions.

Long calculations were avoided by combining Eq. 5 with the relation between A, volume and specific gravity at two different concentrations:

$$A_2 = \frac{A_1V_1d_1}{V_2d_2} = \frac{A_1V_1(1 + aA_1 + bA_1^2 + cA_1^3)}{V_2(1 + aA_2 + bA_2^2 + cA_2^3)}. \tag{7}$$

For the silicate of molecular ratio 1:3.015, this becomes:

$$A_2 = \frac{A_1V_1(1 + 0.0342A_1 + 0.00034A_1^2 + 0.000055A_1^3)}{V_2(1 + 0.0342A_2 + 0.00034A_2^2 + 0.000055A_2^3)} \tag{8}$$

in which V_1, V_2 and A_1 are known quantities. A solution for A_2 may be obtained by a series of approximation steps as follows:

At low concentrations and following the ten-fold dilution technique, $A_2 = 0.100A_1$ very nearly. Substituting in (8),

$$A_2 = \frac{A_1V_1(1 + 0.0342A_1 + 0.00034A_1^2 + 0.000055A_1^3)}{V_2(1 + 0.00342A_1 + 0.0000034A_1^2 + 0.000000055A_1^3)}. \tag{9}$$

In a particular experiment $A_1 = 9.37$ per cent Na_2O and $V_1/V_2 = 0.09776$. Substituting in (9) we obtain 1.2638 for A_2 as a first approximation, and substituting this value in the denominator of (8) we obtain 1.2499 for A_2 as a second approximation. Similarly 1.2504 is obtained as both a third and fourth approximation.

The intersection of the curve of Eq. 5 with that for A_2 and d_2 in Eq. 7 defined the required A_2 and d_2 values.

RESULTS

A. Original Data. Iso-Ratio Curves: pH vs. Per Cent Na_2O

Representative data on sodium hydroxide and silicate systems more complete than earlier work are assembled in Tables II–VII. In Figs. 6–11 are shown the inconsistency between Taylor colorimetric and electrometric pH values plotted on a linear scale against concentration as per cent Na_2O on a logarithmic scale. The curves have been cut off at very low concentrations in some cases where the results become seriously open to question.

B. Changes of pH with Time

The first irregular results were obtained over the intermediate concentration ranges 0.004–0.1 per cent Na_2O for the ratio 1:3.015. The head solution had needed little caustic to shift the ratio from 3.334 and had been stabilized by standing 3 or 4 weeks. Normally a dilution

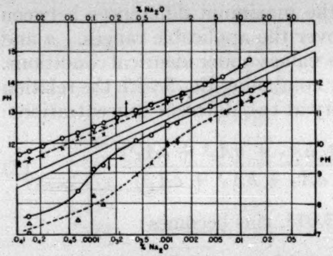


FIG. 6. pH of sodium hydroxide solutions at 20 ± 0.03° C.

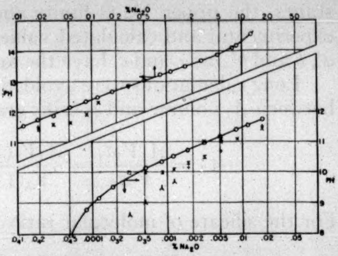


FIG. 7. pH of sodium orthosilicate solutions at 20 ± 0.05° C.

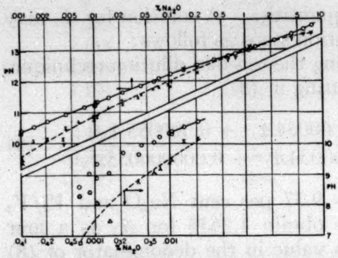


FIG. 8. pH of sodium metasilicate solutions at 20 ± 0.04° C.

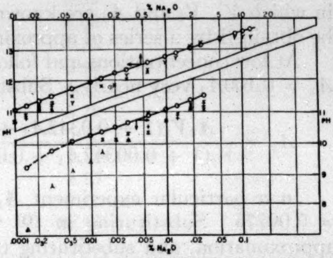


FIG. 9. pH of Na₂O : 1.492 SiO₂ solutions at 20 ± 0.05° C.

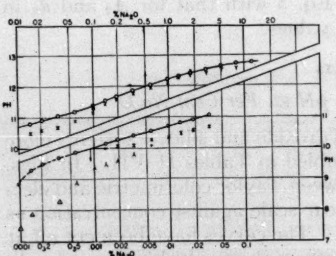


FIG. 10. pH of sodium disilicate solutions at 20 ± 0.05° C.

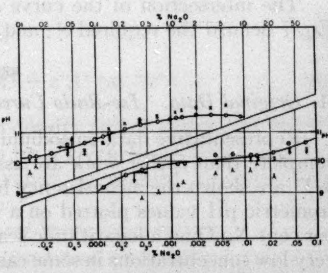


FIG. 11. pH of Na₂O : 3.015 SiO₂ solutions at 20 ± 0.05° C.

(Key to Figs. 6 to 11 inclusive)

Electrometric data—solid line
Colorimetric data—broken line

Acyl Blue	Y	Cresol Red	△
Parazo Orange	×	Bromthymol Blue	⊡
Tolyl Red	+	Acyl Red	S
Phthalein Red	⅄		

.0₄1 stands for .00001; 0₃2 = .0002, etc.

series was carried through in one or two days, but the most erratic points came from solutions which stood three days after intermediate dilutions.

Starting with the same stock solution at 9.370 per cent Na_2O, direct dilutions were made in one step to 1.008, 0.09901, 0.01258, and 0.001204 per cent Na_2O, and pH measurements in the several cases were made as soon as possible after dilution. The measurements were repeated for the four stocks over periods up to three weeks and the pH-time curves are plotted in Fig. 12.

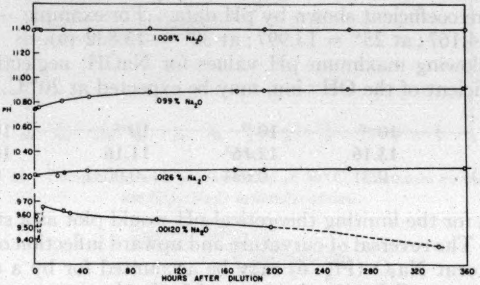


FIG. 12. Effect of aging on the pH of $Na_2O:3.015\ SiO_2$ solutions.

Inspection of data for Fig. 12 shows, at 1.008 per cent Na_2O, over a period of fifteen days an insignificant total variation of only 0.007 pH unit. Measurements were made on different days, with different hydrogen electrodes, with different fillings of the apparatus, with newly formed liquid-liquid junctions, etc.

The behaviors at the three lower concentrations differ not only from that at 1.008 per cent Na_2O but among themselves. The most significant results are probably found at 0.09901 per cent Na_2O where the pH appears to be approaching a maximum value after twenty-one days when the changes are scarcely outside the experimental error. That this curve would probably reach a maximum value is supported by the pH behavior of the next lower concentration, which was actually carried past a maximum emf. at 358.6 hr.

The dependability of results for the lower concentrations was not equal to that for the first two curves. At the lowest concentration, 0.001204 per cent Na_2O, attainment of equilibrium was difficult. The experimental technique which allowed averaging four measurements is of particular value in such cases, but what happens in the first thirty or forty minutes after dilution cannot be learned by the present technique.

DISCUSSION

A. Theoretical Considerations, Temperature Effects, Limiting pH Values

Assuming complete ionization of NaOH, a maximum limiting $[OH]^-$ exists and pH may be computed from

$$pH = \log [OH]^- - \log Kw \qquad (10)$$

where $[OH]^-$ equals normality of NaOH and *Kw* is the ionization constant of water. Density changes affect pH values by only about 0.00005 unit per degree Centigrade, but the value of *Kw* changes rapidly with temperature and may be considered responsible for the considerable temperature coefficient shown by pH data. For example $-\log Kw$ at 20° C. = 14.167; at 25° = 13.997; at 30° = 13.832 (9).

The following maximum pH values for NaOH, neglecting the activity coefficient of the OH^- ion, may be expected at 20° C.:

N	10^{-1}	10^{-2}	10^{-3}	10^{-4}
pH	13.16	12.16	11.16	10.16
% Na_2O	0.31	0.031	0.0031	0.00031

These data for the limiting theoretical pH would plot as a straight line on Fig. 6. The reversal of curvature and upward inflection of the curve past 2 per cent Na_2O (Fig. 6) may be accounted for by a complex of density increase, dielectric change, and ionization constant variation. Although the emf. values were quite reproducible the results should be interpreted as "Apparent pH Values."

B. Comparison of New Data with Published Results

Data which may be converted into terms of pH and concentration of sodium hydroxide or sodium silicate solutions at temperatures comparable with the data of this paper are found in papers by Bogue (10), Hagg (11), Harman (12), and Kuentzel *et al* (13). Bogue used a tenth normal calomel cell as a reference standard while the others all used saturated cells. Hagg and Harman did not convert to pH units. Hagg's emf. data at 20° C. have been converted to pH by Eq. 4 and his concentration data (inverse volume normality $1/N_v$) to per cent Na_2O using density relations derived from International Critical Tables (14) for sodium hydroxide and from Philadelphia Quartz Co. gravity-composition charts in the case of the silicates. Hagg measured the emf. of metasilicate solutions prepared from crystallized $Na_2SiO_3 \cdot 9H_2O$, from fusion of sodium carbonate and silica and from solution of silica in sodium hydroxide. His curves diverge only slightly with concentration change above or below 0.1 per cent Na_2O and reach a maximum difference of 0.07 at 2 per cent Na_2O which appear insignificant from the point of view of constitution. His silicate data are in the best agreement generally with the new data of this paper. A comparison of the results is given in Fig. 13 in which our results have been interpolated to the

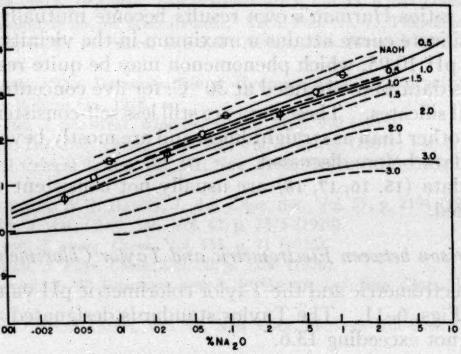


FIG. 13. Comparison with previous data at 20° C. Numbers on curves are $SiO_2:Na_2O$ molecular ratios.

Bacon——solid line
Hagg--------broken line
Kuentzel *et al*—⊖ NaOH
○ $Na_2O:0.447\ SiO_2$
ø $Na_2O:SiO_2$

exact ratios. Hagg's NaOH values are out of line as they exceed not only our own values but the theoretical limiting values in some cases.

% Na_2O	3.1	0.31	0.031	0.0031	0.00031	
pH, Theoretical Maximum	14.16	13.16	12.16	11.16	10.16	
pH, Hagg	—	13.13	12.24	11.19*	—	Read from curves
pH, This Paper	13.89	12.99	12.06	11.08	9.91	Read from curves

* Extrapolated.

The data obtained by Kuentzel *et al* at 25° C. as corrected to 20° C. by the relation $pH_{t_2} = pH_{t_1} + A(t_2 - t_1)$ or calculated directly to 20° from the relation $pH = B + At$, are also shown in Fig. 13. For sodium hydroxide the pH values are lower than given in this paper by 0.03 to 0.09 pH units.[5]

Harman's work done at 25° C. computed and applied liquid-liquid junction corrections. This would increase pH values of orthosilicates or decrease those of tetrasilicates by about 0.10 at $N_w = 2$. At $N_w = 1$ the corrections are insignificant. The pH values have been calculated using Harman's corrected emf. values and the relation pH = (emf. − 0.2515)/0.05911. For metasilicate concentrations above 0.1 per cent

---

[5] The relation adopted by Kuentzel to fix the potential of his calomel cells leads to values about 2 mv. lower with the result that his pH values are about 0.03 higher than would be computed by Eq. 4. The apparent pH differences therefore must be increased by 0.03 in relation to the data of Hagg or this paper, that is, 0.06 to 0.12.

Na_2O, Harman's pH values tend to become inconsistent with his orthosilicate results.

Hagg's and Harman's disilicate curves are sigmoidal and cross, and for higher ratios Harman's own results become mutually inconsistent. The tetrasilicate curve attains a maximum in the vicinity of 2 per cent Na_2O and pH 10.95, which phenomenon may be quite real.

Bogue's data were obtained at 30° C. for five concentrations each of commercial silicates. These data are still less self-consistent, can hardly be plotted other than as straight lines, and are mostly beyond reconciliation with data before discussed.

Other data (**15, 16, 17, 18**) are usually not consistent with the data here reported.

C. Comparison between Electrometric and Taylor Colorimetric pH Values

Both electrometric and the Taylor colorimetric pH values have been plotted in Figs. 6–11. The Taylor standards designated are limited to pH values not exceeding 13.6.

For sodium hydroxide the colorimetric values show a very regular behavior such that a curve can be properly used to correlate values by the several indicators through the whole concentration range. The Taylor values are always lower at given concentrations and this effect increases with dilution.

For sodium orthosilicate the colorimetric data are too irregular to connect the behavior, the sodium metasilicate observations are more regular, but for more siliceous ratios curves were not drawn. In general, colorimetric pH values fall off more rapidly with dilution than the electrometric ones, but the difference diminishes with the more siliceous ratios. For the 1:1.492 and the disilicate ratios there are fairly long ranges in which the agreement is equal to the experimental reproducibility. The deviations at the low concentrations become so serious that below 0.01 per cent Na_2O colorimetric results are hardly worthwhile, even with correction. Except for sodium hydroxide and sodium metasilicate, it is better to estimate the correction from Figs. 6–11 directly than to set up formal relations.

Acknowledgment

The interest and direction given this work by Dr. Wm. Stericker, C. L. Baker and the late Dr. James G. Vail, all of the Chemical Department of the Philadelphia Quartz Company, and the permission to publish these results are gratefully acknowledged.

REFERENCES

(1) W. J. HAMER, private communication, July 12, 1938.

(2) W. J. HAMER AND S. F. ACREE, *Jour. Research NBS*, Vol. 33, p. 87 (1944).

(3) Leeds and Northrup Co., "Standard Conversion Tables, Voltage to pH; Standard Part 1074," Philadelphia, no date.

(4) D. I. HITCHCOCK AND A. C. TAYLOR, *J. Am. Chem. Soc.*, Vol. 60, p. 2710 (1938).
D. A. MACINNES, D. BELCHER, AND T. SHEDLOVSKY, *ibid.*, Vol. 60, p. 1094 (1938).
B. WINGFIELD, W. H. GOSS, W. J. HAMER, AND S. J. ACREE, *ASTM Bull.*, No. 90, p. 15 (1938).
W. J. HAMER AND S. J. ACREE, *Jour. Research NBS*, Vol. 23, p. 647 (1939).
ASTM "Symposium on pH Measurement," Tech. Pub. No. 73, Philadelphia, Pa.

(5) W. J. HAMER, G. D. PINCHING, AND S. F. ACREE, *Jour. Research NBS*, Vol. 36, p. 47 (1946).
W. M. CLARK, "The Determination of Hydrogen Ions," Third edition, Baltimore, Williams & Wilkins Co., 1928.

(6) E. J. KING AND H. STANTIAL, *Biochem. J.*, Vol. 27, p. 990 (1933).

(7) J. E. S. HAN, *Ind. Eng. Chem.*, *Anal. Ed.*, Vol. 9, p. 140 (1937); Vol. 6, p. 209 (1934).

(8) International Critical Tables, Vol. III, New York, McGraw-Hill Book Co., Inc., 1928, p. 79.

(9) H. S. HARNED AND W. J. HAMER, *J. Am. Chem. Soc.*, Vol. 55, p. 2194 (1933).

(10) R. H. BOGUE, *J. Am. Chem. Soc.*, Vol. 42, p. 2575 (1920).

(11) GUNNAR HAGG, *Z. anorg. Chem.*, Vol. 155, p. 21 (1926).

(12) R. W. HARMAN, *J. Phys. Chem.*, Vol. 30, p. 1100 (1926).

(13) L. E. KUENTZEL, J. W. HENSLEY, AND L. R. BACON, *Ind. Eng. Chem.*, Vol. 35, p. 1286 (1943).

(14) International Critical Tables, Vol. VI, New York, McGraw-Hill Book Co., Inc., 1928, p. 152.

(15) L. THOMPSON, Master's Thesis, Worcester Polytechnic Institute, 1923.

(16) F. D. SNELL, *Ind. Eng. Chem.*, Vol. 24, p. 76 (1932).

(17) A. T. WILLIAMSON AND W. G. OAKES, Imperical Chemical Industries, Ltd., private communication, 1920.

(18) W. D. TREADWELL AND W. WIELAND, *Helv. Chim. Acta*, Vol. 13, p. 842 (1930).

Gas-Insulated Transformers.—Future transformers may be lighter, quieter, and safer as a result of a new manufacturing development which also may lead to the elimination of liquids as power transformer insulations and coolants, according to W. S. Ginn, general manager of the General Electric Company's Power Transformer Department.

The development, a joint project with the Consolidated Edison Company of New York, involves the first application of a high-dielectric gaseous insulation to high voltage power transformers.

Mr. Ginn said it foretells the eventual displacement of oil- and askarel-filled transformer construction by gas-filled, dry-type construction. The new dry-type units filled with sulfur hexafluoride gas will be suitable for both indoor and outdoor installation.

Two of the new type units, rated 2000 kva, 69 kv, are now on the General Electric drawing boards. When completed, the transformers will be operated for field test experience by the Consolidated Edison Company. With the benefit of field testing, the new gas-filled construction can conceivably be extended to considerably higher voltages, Mr. Ginn said.

One of the main advantages of the new transformer will be its safer operating characteristics without the complications of present liquid dielectrics. Mr. Ginn said that G.E. introduced Pyranol * dielectric, a non-flammable liquid, in 1932 to reduce the fire hazard in commercial and industrial transformer installations. However, Pyranol insulated power transformers are limited to ratings of 46,000 volts and below.

Because pressurized gas is a much poorer transmitter of sound than liquid, the new transformers are expected to have improved sound characteristics compared with those manufactured currently. Furthermore, the omission of oil and the reduced core and coil dimensions which may result from the new gaseous insulation should produce transformers which are lighter in weight than similarly rated units of conventional design, the general manager added.

Lynn Wetherill, manager of engineering, of General Electric's Power Transformer Department, explained that prototype designs of the new transformer have been under development for more than two years. Until now, he said, dry-type transformer construction has been limited to about 15 kv. While many existing units have sealed nitrogen-filled tanks, the main purpose of the nitrogen is to exclude oxygen and water vapor rather than to obtain any improvement in the dielectric and cooling properties. Laboratory investigations indicated that sulfur hexafluoride under moderate pressure offers considerable promise both as a dielectric and as a coolant. The Consolidated Edison transformers will utilize a gas-to-air heat exchanger to cool the gas after it is pumped through the core and coil assembly.

Mr. Ginn suggested that the practical application of gas as a high voltage transformer insulation and coolant is probably the most radical change in transformer design concepts since the introduction in 1903 of non-aging silicon core steel which more than doubled transformer life.

---

* Registered trademark of the General Electric Company.

THREE-DIMENSIONAL STRESS FUNCTIONS

BY

H. L. LANGHAAR[1] AND M. STIPPES[1]

SUMMARY

Four types of stress functions are known for solving elasticity problems: the components of the displacement vector, the components of the Galerkin vector, the Maxwell stress functions, and the Morera stress functions. For problems with stress type boundary conditions, the Maxwell stress functions are, in many respects, the simplest to use, but they lack the simple transformation properties of vectors. It was shown by C. Weber (**1**)[2] that the Maxwell and Morera functions supplement each other, and that together they are the components of a second order symmetric Cartesian tensor.

In this paper, the compatibility equations for an isotropic Hookean body that is subjected to boundary stresses and temperature gradients are developed in terms of the Maxwell stress functions, and their general solution is presented for steady temperature fields. It is shown that, when the complementary energy of a homogeneous body with arbitrary elastic properties is expressed in terms of the components of the Maxwell-Morera tensor, the Euler equations for the integral of the complementary energy density are the complete set of compatibility equations in terms of the stress components. The Maxwell-Morera tensor is generalized, so that it represents the general solution of the equilibrium equations in any curvilinear coordinates. As an application, the general solution of the equilibrium equations in cylindrical coordinates is derived.

MAXWELL'S STRESS FUNCTIONS

Clerk Maxwell observed (**2**) that, if body force is absent, the differential equations of equilibrium of a deformable body that is referred to rectangular coordinates (x, y, z) are identically satisfied by

$$\sigma_x = B_{zz} + C_{yy} \quad \sigma_y = C_{xx} + A_{zz} \quad \sigma_z = A_{yy} + B_{xx} \tag{1a}$$

$$\tau_{yz} = -A_{yz} \qquad \tau_{zx} = -B_{zx} \qquad \tau_{xy} = -C_{xy} \tag{1b}$$

in which A, B, C are arbitrary functions of x, y, z. The subscripts on A, B, C denote partial derivatives.

Furthermore, any solution of the equilibrium equations can be represented by Eqs. 1. For, if $\sigma_x, \cdots, \tau_{xy}$ are given functions of x, y, z that satisfy the equilibrium equations, the functions A, B, C are determined within certain arbitrary additive functions by Eqs. 1*b*. If the shearing stresses are eliminated from the equilibrium equations by Eqs. 1*b*, the resulting equations may be integrated, and Eqs. 1*a* are obtained, except for certain additive functions which may be set equal to zero in virtue of the arbitrariness in the functions A, B, C. Accordingly, Eqs. 1 are a representation of the general solution of the equilibrium equations.

---

[1] Department of Theoretical and Applied Mechanics, University of Illinois, Urbana, Ill.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

The functions A, B, C are not unique, for if the stresses are all zero, Eqs. 1 have the following solution

$$A_0 = \frac{\partial^2}{\partial x^2}\left[Q_2(z, x) - Q_3(x, y)\right] \tag{2}$$

$$B_0 = \frac{\partial^2}{\partial y^2}\left[Q_3(x, y) - Q_1(y, z)\right] \tag{3}$$

$$C_0 = \frac{\partial^2}{\partial z^2}\left[Q_1(y, z) - Q_2(z, x)\right] \tag{4}$$

in which Q_1, Q_2, Q_3 are arbitrary functions. Any functions of the type A_0, B_0, C_0 may be added to A, B, C without alteration of the stresses. In particular, A_0, B_0, C_0 may be arbitrary functions of x, y, z, respectively.

COMPATIBILITY CONDITIONS

The compatibility equations in the small displacement theory of an isotropic Hookean body that is subjected to boundary stresses and temperature gradients are

$$(1 + \nu)\nabla^2\sigma_x + \frac{\partial^2\theta}{\partial x^2} + E\alpha\left[\frac{1 + \nu}{1 - \nu}\nabla^2 T + \frac{\partial^2 T}{\partial x^2}\right] = 0, \cdots, \cdots; \tag{5a}$$

$$(1 + \nu)\nabla^2\tau_{yz} + \frac{\partial^2\theta}{\partial y\partial z} + E\alpha\frac{\partial^2 T}{\partial y\partial z} = 0, \cdots, \cdots; \tag{5b}$$

$$\theta = \sigma_x + \sigma_y + \sigma_z \tag{5c}$$

in which $T(x, y, z)$ is the temperature, and $E\alpha$ is the product of Young's modulus and the coefficient of thermal expansion. Equations 5 may be derived by substituting the expressions for the strains in terms of the stresses and the temperature (3, p. 421) into the strain compatibility equations (3, p. 230) and simplifying the resulting equations with the aid of the equilibrium equations. This method is outlined in (3, p. 231).

Attention will be restricted to temperature distributions that are independent of the time. Then, by the theory of heat conduction, $\nabla^2 T = 0$ and, by adding Eqs. 5a together, we obtain $\nabla^2\theta = 0$. Hence $\nabla^2\bar{\theta} = 0$ where

$$\bar{\theta} = E\alpha T + \theta. \tag{6}$$

Substituting Eqs. 1*a* into the first of Eqs. 5*a* and writing $\bar{\theta}_{xx}$ in the form $-\bar{\theta}_{yy} - \bar{\theta}_{zz}$, we obtain

$$\frac{\partial^2}{\partial z^2}\left[(1+\nu)\nabla^2 B - \bar{\theta}\right] + \frac{\partial^2}{\partial y^2}\left[(1+\nu)\nabla^2 C - \bar{\theta}\right] = 0. \tag{7a}$$

Two corresponding equations are obtained by cyclic permutation of the variables. Also, substitution of Eqs. 1*b* into Eqs. 5*b* yields

$$\frac{\partial^2}{\partial y \partial z}\left[(1+\nu)\nabla^2 A - \bar{\theta}\right] = 0, \cdots, \cdots. \tag{7b}$$

Equations 7 are of the same form as the differential equations that determine A_0, B_0, C_0. Consequently, the expressions

$$(1+\nu)\nabla^2 A - \bar{\theta}, \quad (1+\nu)\nabla^2 B - \bar{\theta}, \quad (1+\nu)\nabla^2 C - \bar{\theta}$$

are functions of the type A_0, B_0, C_0 (Eqs. 2, 3, 4). By adding to A, B, C appropriate functions $A_0{}^*$, $B_0{}^*$, $C_0{}^*$ that do not alter the stresses, we may cancel the terms on the right side of the equations. Accordingly, when the class of arbitrary functions A_0, B_0, C_0 is restricted appropriately, the compatibility equations for the functions A, B, C are [3]

$$\nabla^2 A = \nabla^2 B = \nabla^2 C = \frac{\bar{\theta}}{1+\nu}. \tag{8}$$

By Eqs. 1*a*

$$\theta = \nabla^2 A + \nabla^2 B + \nabla^2 C - (A_{xx} + B_{yy} + C_{zz}).$$

With Eqs. 6 and 8, this yields

$$\theta = \frac{1+\nu}{2-\nu}(A_{xx} + B_{yy} + C_{zz}) - \frac{3E\alpha T}{2-\nu}. \tag{9}$$

Consequently, Eq. 8 yields

$$\nabla^2 A = \nabla^2 B = \nabla^2 C = \frac{A_{xx} + B_{yy} + C_{zz}}{2-\nu} - \frac{E\alpha T}{2-\nu}. \tag{10}$$

Let $\phi(x, y, z)$ be any particular integral of the Poisson equation

$$\nabla^2 \phi = -\frac{E\alpha T}{1-\nu}. \tag{11}$$

---

[3] These equations were obtained by Clerk Maxwell in the previously cited paper by a different method that did not include thermal effects. However, he did not derive the general solution of the equations.

Let

$$A = A' + \phi, \quad B = B' + \phi, \quad C = C' + \phi. \tag{12}$$

Then Eq. 10 reduces to

$$\nabla^2 A' = \nabla^2 B' = \nabla^2 C' = \frac{A_{xx}' + B_{yy}' + C_{zz}'}{2 - \nu}. \tag{13}$$

Thus thermal effects are separated from the problem.

From the form of Eqs. 13, we observe that $A' - C'$ and $B' - C'$ are arbitrary harmonic functions $\bar{H}_1(x, y, z)$, $\bar{H}_2(x, y, z)$. Utilizing this fact, we obtain, for C', the following differential equation

$$(1 - \nu)\nabla^2 C' = \frac{\partial^2 \bar{H}_1}{\partial x^2} + \frac{\partial^2 \bar{H}_2}{\partial y^2} \tag{14}$$

of which the general solution is

$$C'(x, y, z) = \frac{x}{2(1 - \nu)} \frac{\partial \bar{H}_1}{\partial x} + \frac{y}{2(1 - \nu)} \frac{\partial \bar{H}_2}{\partial y} + \frac{1}{1 - \nu} \bar{H}_3(x, y, z),$$

where, again $\bar{H}_3(x, y, z)$ is an arbitrary harmonic function. The solutions A', B', C', however, so obtained, are not symmetric in appearance. If we repeat the process using B' as the parameter function and then A' as the parameter function and add the results obtained for each function, we obtain

$$A = \frac{1}{2(1 - \nu)}\left[x \frac{\partial H_1}{\partial x} + y \frac{\partial H_2}{\partial y} + z \frac{\partial H_3}{\partial z}\right] - H_2 - H_3 + \phi, \cdots, \cdots \tag{15}$$

where H_1, H_2, H_3 are arbitrary harmonic functions. The remaining expressions are obtained by cyclically permuting the functions A, B, C; H_1, H_2, H_3; and x, y, z. Equations 15 are a general solution of the equations of elasticity with steady thermal effects.

From the first of Eqs. 8 we obtain

$$(1 + \nu)(A_{xx} - B_{xx} - C_{xx} + \sigma_y + \sigma_z) - (\sigma_x + \sigma_y + \sigma_z) - E\alpha T = 0$$

or

$$\frac{1 + \nu}{E}(A_{xx} - B_{xx} - C_{xx}) - \epsilon_x = 0$$

so that

$$\frac{\partial u}{\partial x} = \frac{1 + \nu}{E}(A_{xx} - B_{xx} - C_{xx}), \cdots, \cdots.$$

These equations may be integrated, and, in view of the fact that the shearing strains must conform to Eqs. 1*b*, the additive functions can only represent a rigid body displacement. Consequently, *aside from a rigid body displacement,*

$$
\begin{aligned}
u &= \frac{1+\nu}{E}\,(A_x - B_x - C_x) \\
v &= \frac{1+\nu}{E}\,(B_y - C_y - A_y) \qquad (16)^{4} \\
w &= \frac{1+\nu}{E}\,(C_z - A_z - B_z).
\end{aligned}
$$

Thus, the functions A, B, C determine the displacements directly. However, Eqs. 16 are valid only if the arbitrary functions A_0, B_0, C_0 are restricted by Eq. 8.

A general solution of the displacement equations of equilibrium (without thermal effects) was derived by Papkovich (3, p. 235). In view of Eqs. 16, the functions H_1, H_2, H_3 are related to Papkovich's functions ϕ_1, ϕ_2, ϕ_3 by the equations

$$\phi_1 = \frac{2(1+\nu)}{E}\frac{\partial H_1}{\partial x}, \quad \phi_2 = \frac{2(1+\nu)}{E}\frac{\partial H_2}{\partial y}, \quad \phi_3 = \frac{2(1+\nu)}{E}\frac{\partial H_3}{\partial z}. \qquad (17)$$

These relationships verify that all possible systems of stress are represented by Eqs. 15.

Since to any displacement vector, there corresponds a Galerkin vector (4) and vice versa, Eq. 16 relates the Maxwell functions to the Galerkin vector.

To reduce the stress type boundary conditions to a convenient form, let the bounding surface of the elastic solid be given by equations of the type

$$x = x(\alpha, \beta), \quad y = y(\alpha, \beta), \quad z = z(\alpha, \beta)$$

and let g be the determinant of the metric tensor on this surface. (It is assumed here that the second derivatives of x, y, z with respect to α and β exist.) It is then possible to reduce an equation of the type

$$\sigma_x l + \tau_{xy} m + \tau_{xz} n = X(\alpha, \beta)$$

to the form

$$
\begin{aligned}
\frac{\partial}{\partial\beta}\left(\frac{\partial y}{\partial\alpha}\frac{\partial B}{\partial z}\right) - \frac{\partial}{\partial\alpha}\left(\frac{\partial y}{\partial\beta}\frac{\partial B}{\partial z}\right) + \frac{\partial}{\partial\alpha}\left(\frac{\partial z}{\partial\beta}\frac{\partial C}{\partial y}\right) \\
- \frac{\partial}{\partial\beta}\left(\frac{\partial z}{\partial\alpha}\frac{\partial C}{\partial y}\right) = \sqrt{g}X(\alpha, \beta). \qquad (18)
\end{aligned}
$$

The remaining two stress boundary conditions are again obtained by permuting A, B, C; X, Y, Z; x, y, z.

---

[4] **These equations were presented by Maxwell, *loc. cit.***

MORERA'S STRESS FUNCTIONS

G. Morera (5) observed that, if body force is absent, the differential equations of equilibrium are satisfied identically by

$$\sigma_x = L_{yz}, \quad \sigma_y = M_{zx}, \quad \sigma_z = N_{xy} \tag{19a}$$

$$\tau_{yz} = -\tfrac{1}{2}(M_y + N_z - L_x)_x, \quad \tau_{zx} = -\tfrac{1}{2}(N_z + L_x - M_y)_y,$$
$$\tau_{xy} = -\tfrac{1}{2}(L_x + M_y - N_z)_z \tag{19b}$$

in which L, M, N are arbitrary functions of (x, y, z). Furthermore, any solution of the equilibrium equations can be represented by Eq. 19. The proof of this statement is the same as for the Maxwell stress functions. Accordingly, Eqs. 19 are another representation of the general solution of the equilibrium equations.

The functions L, M, N are not unique, for if the stresses are all zero, Eqs. 19 have the following solution

$$\begin{aligned} L_0 &= \frac{\partial}{\partial z} R_2(z, x) + \frac{\partial}{\partial y} R_3(x, y) \\ M_0 &= \frac{\partial}{\partial x} R_3(x, y) + \frac{\partial}{\partial z} R_1(y, z) \\ N_0 &= \frac{\partial}{\partial y} R_1(y, z) + \frac{\partial}{\partial x} R_2(z, x) \end{aligned} \tag{20}$$

in which R_1, R_2, R_3 are arbitrary functions. Any functions of the type L_0, M_0, N_0 may be added to L, M, N, respectively, without altering the stresses.

The compatibility equations (5a, 5b) do not readily submit to a treatment of the type presented for the derivation of Eqs. 15, if the Morera functions are used. However, it is still possible to present expressions for L, M, N as combinations of derivatives of harmonic functions by the following method. A given state of stress may be represented either by Maxwell's or Morera's stress functions; and, for the functions A, B, C and L, M, N to represent the same state of stress, the following relations exist between the two sets of functions:

$$B_{zz} + C_{yy} = L_{yz}, \cdots, \cdots;$$
$$-A_{xy} = -\tfrac{1}{2}(L_x + M_y - N_z)_z, \cdots, \cdots. \tag{21}$$

If thermal stresses are absent, we may choose the arbitrary functions in the description (15) in the form

$$H_1 = \frac{\partial^3 \phi_1}{\partial x \partial y \partial z}, \quad H_2 = \frac{\partial^3 \phi_2}{\partial x \partial y \partial z}, \quad H_3 = \frac{\partial^3 \phi_3}{\partial x \partial y \partial z} \tag{22}$$

where ϕ_1, ϕ_2, ϕ_3 are arbitrary harmonic functions. With these new functions, we have

$$A = \frac{\partial^3}{\partial x \partial y \partial z}[\mathfrak{M} - \phi_2 - \phi_3], \quad B = \frac{\partial^3}{\partial x \partial y \partial z}[\mathfrak{M} - \phi_3 - \phi_1],$$
$$C = \frac{\partial^3}{\partial x \partial y \partial z}[\mathfrak{M} - \phi_1 - \phi_2] \qquad (23)$$

where

$$\mathfrak{M} = \frac{1}{2(1-\nu)}\left[x\frac{\partial \phi_1}{\partial x} + y\frac{\partial \phi_2}{\partial y} + z\frac{\partial \phi_3}{\partial z} - \phi_1 - \phi_2 - \phi_3\right]. \qquad (24)$$

Then, from Eq. 21

$$\begin{aligned} L &= \frac{\partial}{\partial x}[\nabla^2\mathfrak{M} - \mathfrak{M}_{xx} + \phi_{1xx} - \phi_{2yy} - \phi_{3zz}] \\ M &= \frac{\partial}{\partial y}[\nabla^2\mathfrak{M} - \mathfrak{M}_{yy} + \phi_{2yy} - \phi_{3zz} - \phi_{1xx}] \qquad (25) \\ N &= \frac{\partial}{\partial z}[\nabla^2\mathfrak{M} - \mathfrak{M}_{zz} + \phi_{3zz} - \phi_{1xx} - \phi_{2yy}]. \end{aligned}$$

Hooke's law now provides

$$\epsilon_x = -\frac{\nu}{E}\frac{\partial^3}{\partial x \partial y \partial z}[2\nabla^2\mathfrak{M} - \phi_{1xx} - \phi_{2yy} - \phi_{3zz}] + \frac{1+\nu}{E}\frac{\partial^3}{\partial x \partial y \partial z}[\nabla^2\mathfrak{M} - \mathfrak{M}_{xx} + \phi_{1xx} - \phi_{2yy} - \phi_{3zz}]$$

and hence

$$u = -\frac{\nu}{E}\frac{\partial^2}{\partial y \partial z}[2\nabla^2\mathfrak{M} - \phi_{1xx} - \phi_{2yy} - \phi_{3zz}] + \frac{1+\nu}{E}\frac{\partial^2}{\partial y \partial z}[\nabla^2\mathfrak{M} - \mathfrak{M}_{xx} + \phi_{1xx} - \phi_{2yy} - \phi_{3zz}] \qquad (26)$$

where again the functions of integration vanish.

By combining the Maxwell functions and the Morera functions, we obtain the following general solution of the equilibrium equations

$$\begin{aligned} \sigma_x &= B_{zz} + C_{yy} - 2L_{yz} & \tau_{yz} &= -A_{yz} + (M_y + N_z - L_x)_x \\ \sigma_y &= C_{xx} + A_{zz} - 2M_{zx} & \tau_{zx} &= -B_{zx} + (N_z + L_x - M_y)_y \qquad (27) \\ \sigma_z &= A_{yy} + B_{xx} - 2N_{xy} & \tau_{xy} &= -C_{xy} + (L_x + M_y - N_z)_z \end{aligned}$$

in which A, B, C, L, M, N are arbitrary functions. (The Morera functions have here been multiplied by -2. The reason for this will become apparent upon reading of the section on the Maxwell-Morera tensor.) It may be verified by direct substitution that Eqs. 27 satisfy the differential equations of equilibrium identically. Conversely, any solution of the equilibrium equations is represented by Eqs. 27; indeed, with $L = M = N = 0$ or with $A = B = C = 0$ as was shown previously.

VARIATIONAL THEOREM ON COMPATIBILITY

If a homogeneous body is elastic, in the sense that the work of deformation is fully recoverable, and if the geometric approximations of small displacement theory are used, there exists [5] (6) a complementary energy density function $\Omega(\sigma_x, \cdots, \tau_{xy})$ such that the strains are given by

$$\begin{aligned} \epsilon_x &= \frac{\partial\Omega}{\partial\sigma_x} \quad & \epsilon_y &= \frac{\partial\Omega}{\partial\sigma_y} \quad & \epsilon_z &= \frac{\partial\Omega}{\partial\sigma_z} \\ \gamma_{yz} &= \frac{\partial\Omega}{\partial\tau_{yz}} \quad & \gamma_{zx} &= \frac{\partial\Omega}{\partial\tau_{zx}} \quad & \gamma_{xy} &= \frac{\partial\Omega}{\partial\tau_{xy}}. \end{aligned} \tag{28}$$

If the stress-strain relations are linear, Ω is identical to the strain energy density.

The complementary energy of the body is

$$\Upsilon = \iiint \Omega dxdydz.$$

In view of Eqs. 27, Υ may be regarded as a functional of the variables A, B, C, L, M, N. Then the Euler equations for the integral Υ are

$$\begin{aligned} &\frac{\partial^2}{\partial x^2}\frac{\partial\Omega}{\partial A_{xx}} + \frac{\partial^2}{\partial y^2}\frac{\partial\Omega}{\partial A_{yy}} + \frac{\partial^2}{\partial z^2}\frac{\partial\Omega}{\partial A_{zz}} \\ &\qquad + \frac{\partial^2}{\partial y\partial z}\frac{\partial\Omega}{\partial A_{yz}} + \frac{\partial^2}{\partial z\partial x}\frac{\partial\Omega}{\partial A_{zx}} + \frac{\partial^2}{\partial x\partial y}\frac{\partial\Omega}{\partial A_{xy}} = 0 \end{aligned} \tag{29}$$

with five similar equations corresponding to the functions B, C, L, M, N. In view of Eqs. 27 and 28

$$\frac{\partial\Omega}{\partial A_{xx}} = 0, \quad \frac{\partial\Omega}{\partial A_{yy}} = \frac{\partial\Omega}{\partial\sigma_z} = \epsilon_z$$

$$\frac{\partial\Omega}{\partial A_{zz}} = \frac{\partial\Omega}{\partial\sigma_y} = \epsilon_y, \quad \frac{\partial\Omega}{\partial A_{yz}} = -\frac{\partial\Omega}{\partial\tau_{yz}} = -\gamma_{yz}, \quad \frac{\partial\Omega}{\partial A_{zx}} = \frac{\partial\Omega}{\partial A_{yz}} = 0.$$

---

[5] Thermal stresses are not included in this discussion.

Consequently, Eq. 29 yields

$$\frac{\partial^2 \epsilon_z}{\partial y^2} + \frac{\partial^2 \epsilon_y}{\partial z^2} - \frac{\partial^2 \gamma_{yz}}{\partial y \partial z} = 0.$$

This is one of the compatibility equations for the strains (3, p. 230). Similarly, the Euler equations for the functions B, C, L, M, N yield the other five compatibility equations. *Accordingly, the Euler equations for the complementary energy integral are the complete set of compatibility equations, provided that the complementary energy density is expressed in terms of the functions A, B, C, L, M, N.* This conclusion is useful for deriving the compatibility equations for a material with anisotropic or non-linear stress-strain properties, since the equations are obtained immediately in terms of the stresses, though not necessarily in the canonical form of Beltrami and Michell. For example, for an isotropic Hookean body, the Euler equations corresponding to L, M, N yield Eqs. 5*b* directly, whereas the Euler equations corresponding to A, B, C yield equations that are easily shown to be equivalent to Eqs. 5*a*.

Our conclusion is an obvious consequence of the known theorem that a stationary value of the complementary energy in the class of stresses that satisfy the equilibrium equations and the forced boundary conditions is sufficient to insure the compatibility equations. It is surprising however, that the Euler equations for Υ provide only three of the compatibility equations if Υ is expressed in terms of A, B, C alone (or L, M, N alone), since it has been proved that A, B, C (or L, M, N) provide the general solution to the equilibrium equations. Utilizing the Maxwell stress functions (with no temperature effects), it may be shown that the three equations of the type (5*a*) imply that the expressions

$$(1+\nu)\nabla^2\tau_{xy} + \frac{\partial^2\theta}{\partial x \partial y}, \quad (1+\nu)\nabla^2\tau_{yz} + \frac{\partial^2\theta}{\partial y \partial z}, \quad (1+\nu)\nabla^2\tau_{zx} + \frac{\partial^2\theta}{\partial z \partial x}$$

can only be functions, which, when regarded as another set of Maxwell stress functions, give rise to zero normal stress. The explanation, then, apparently lies in the fact that the Euler equations so obtained must be supplemented by the natural boundary conditions to insure stationary complementary energy. The natural boundary conditions are necessarily equivalent to the three missing compatibility equations, but a detailed analysis of this point is difficult.

THE MAXWELL-MORERA TENSOR

Let g_{ij} be the metric tensor corresponding to arbitrary coordinates x^1, x^2, x^3 in Euclidean space, and let g be the determinant of the matrix (g_{ij}). An absolute antisymmetric tensor e^{ijk} is defined by

$$
\begin{aligned}
e^{123} &= e^{231} = e^{312} = 1/\sqrt{g} \\
e^{213} &= e^{321} = e^{132} = -\,1/\sqrt{g} \\
e^{ijk} &= 0, \quad \text{if any two indices are equal.}
\end{aligned}
$$

Let τ^{ij} be the stress tensor. If T_{ij} is an arbitrary symmetric tensor, the equilibrium equations $\tau^{ij}{}_{,j} = 0$ are satisfied identically by

$$\tau^{ij} = e^{imn}e^{jrs}T_{mr,ns}. \tag{30}$$

In cartesian coordinates, the components of the stress tensor appear as Eqs. 27 and therefore, by Eqs. 23 and 25,

$$
\begin{aligned}
T_{11} = A &= \frac{\partial^3}{\partial x \partial y \partial z}\left[\mathfrak{M} - \phi_2 - \phi_3\right], \cdots, \cdots; \\
T_{23} = L &= \frac{\partial}{\partial x}\left[\nabla^2 \mathfrak{M} - \mathfrak{M}_{xx} + \phi_{1xx} - \phi_{2yy} - \phi_{3zz}\right], \cdots, \cdots.
\end{aligned}
$$

These values of T_{ij} satisfy identically the compatibility equations in cartesian coordinates. With the general solution of the compatibility equations in cartesian coordinates, we may transform T_{ij} into any coordinate system, and consequently, we have the general solution of the equations of elasticity in arbitrary coordinate systems. The same can be said for the Papkovich functions. It is evident that this solution contains less arbitrariness than the Galerkin vector since here we have three arbitrary harmonic functions whereas the Galerkin vector, in general, contains nine arbitrary harmonic functions.

As an example, consider these results in cylindrical coordinates (r, θ, z). The stress vector on a surface element with covariant unit normal n_j is $\tau^{ij}n_j$. Accordingly, if α_i, β_i, γ_i are unit vectors that are respectively tangent to the r, θ, z coordinate lines, the nontensorial stress components (3, p. 306) are

$$
\begin{aligned}
\sigma_r &= \tau^{ij}\alpha_i\alpha_j & \sigma_\theta &= \tau^{ij}\beta_i\beta_j & \sigma_z &= \tau^{ij}\gamma_i\gamma_j \\
\tau_{\theta z} &= \tau^{ij}\beta_i\gamma_j & \tau_{zr} &= \tau^{ij}\gamma_i\alpha_j & \tau_{r\theta} &= \tau^{ij}\alpha_i\beta_j.
\end{aligned}
$$

Consequently, since $\alpha_i = (1, 0, 0)$, $\beta_i = (0, r, 0)$, $\gamma_i = (0, 0, 1)$

$$
\begin{aligned}
\sigma_r &= \tau^{11} & \sigma_\theta &= r^2\tau^{22} & \sigma_z &= \tau^{33} \\
\tau_{\theta z} &= r\tau^{23} & \tau_{zr} &= \tau^{31} & \tau_{r\theta} &= r\tau^{12}.
\end{aligned}
$$

One then obtains from Eq. 30

$$\sigma_r = \frac{1}{r^2}\frac{\partial^2 T_{22}}{\partial z^2} + \frac{1}{r^2}\frac{\partial^2 T_{33}}{\partial \theta^2} + \frac{1}{r}\frac{\partial T_{33}}{\partial r} - \frac{2}{r^2}\frac{\partial^2 T_{23}}{\partial \theta \partial z} - \frac{2}{r}\frac{\partial T_{31}}{\partial z}$$

$$\sigma_\theta = \frac{\partial^2 T_{33}}{\partial r^2} + \frac{\partial^2 T_{11}}{\partial z^2} - 2\frac{\partial^2 T_{31}}{\partial r \partial z}$$

$$\sigma_z = \frac{1}{r^2}\frac{\partial^2 T_{11}}{\partial \theta^2} + \frac{1}{r^2}\frac{\partial^2 T_{22}}{\partial r^2} - \frac{2}{r^2}\frac{\partial^2 T_{12}}{\partial r \partial \theta} - \frac{1}{r}\frac{\partial T_{11}}{\partial r} - \frac{2}{r^3}\frac{\partial T_{22}}{\partial r} + \frac{2}{r^4} T_{22}$$

$$\tau_{\theta z} = -\frac{1}{r}\frac{\partial^2 T_{11}}{\partial \theta \partial z} + \frac{1}{r}\frac{\partial^2 T_{12}}{\partial r \partial z} + \frac{1}{r}\frac{\partial^2 T_{31}}{\partial r \partial \theta} - \frac{1}{r}\frac{\partial^2 T_{23}}{\partial r^2} + \frac{1}{r^2}\frac{\partial T_{12}}{\partial z} - \frac{1}{r^2}\frac{\partial T_{31}}{\partial \theta} + \frac{1}{r^2}\frac{\partial T_{23}}{\partial r} \qquad (31)$$

$$\tau_{zr} = -\frac{1}{r^2}\frac{\partial^2 T_{22}}{\partial r \partial z} + \frac{1}{r^2}\frac{\partial^2 T_{23}}{\partial r \partial \theta} + \frac{1}{r^2}\frac{\partial^2 T_{12}}{\partial \theta \partial z} - \frac{1}{r^2}\frac{\partial^2 T_{31}}{\partial \theta^2} + \frac{1}{r^3}\frac{\partial T_{22}}{\partial z} + \frac{1}{r}\frac{\partial T_{11}}{\partial z}$$

$$\tau_{r\theta} = -\frac{1}{r}\frac{\partial^2 T_{33}}{\partial r \partial \theta} + \frac{1}{r}\frac{\partial^2 T_{31}}{\partial \theta \partial z} + \frac{1}{r}\frac{\partial^2 T_{23}}{\partial r \partial z} - \frac{1}{r}\frac{\partial^2 T_{12}}{\partial z^2} + \frac{1}{r^2}\frac{\partial T_{33}}{\partial \theta} - \frac{2}{r^2}\frac{\partial T_{23}}{\partial z}.$$

(These equations may be simplified slightly by the substitution $T_{22} = r^2 T_{22}'$.) The equations of equilibrium are (3, p. 306)

$$\frac{\partial \sigma_r}{\partial r} + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} = 0$$

$$\frac{\partial \tau_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_\theta}{\partial \theta} + \frac{\partial \tau_{\theta z}}{\partial z} + \frac{2\tau_{r\theta}}{r} = 0$$

$$\frac{\partial \tau_{zr}}{\partial r} + \frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\partial \sigma_z}{\partial z} + \frac{\tau_{rz}}{r} = 0.$$

These equations are satisfied identically by Eq. 31.

By setting $T_{23} = T_{31} = T_{12} = 0$, we obtain a solution of the equilibrium equations in terms of three functions T_{11}, T_{22}, T_{33}, which are analogous to the Maxwell functions. By setting $T_{11} = T_{22} = T_{33} = 0$, we obtain a solution in terms of three functions T_{12}, T_{13}, T_{23} which are analogous to the Morera functions.

A sufficient condition that the stresses be independent of θ is that the functions T_{ij} be independent of θ. Then a sufficient condition that $\tau_{r\theta} = \tau_{\theta z} = 0$ is $T_{12} = T_{23} = 0$. The general axially symmetrical solution of the equilibrium equations for which $\tau_{r\theta} = \tau_{\theta z} = 0$ is obtained by setting

$$T_{11} = \frac{T_{22}}{r^2} = F(r, z), \quad T_{33} = H(r, z)$$

$$T_{23} = T_{31} = T_{12} = 0.$$

Then, by Eq. 31,

$$\begin{aligned}\sigma_r &= F_{zz} + \frac{1}{r} H_r \\ \sigma_\theta &= F_{zz} + H_{rr} \\ \sigma_z &= F_{rr} + \frac{1}{r} F_r \\ \tau_{rz} &= - F_{rz}.\end{aligned} \tag{32}$$

It is easily verified that, if σ_r, σ_θ, σ_z, τ_{rz} are given functions of r and z that satisfy the equilibrium equations, and if $\tau_{r\theta} = \tau_{\theta z} = 0$, there exist functions $F(r, z)$, $H(r, z)$ that satisfy Eqs. 32. The proof is analogous to that for the existence of the Maxwell functions.

REFERENCES

(1) C. WEBER, "Spannungsfunktionen des dreidimensionalen Kontinuums," *Zeitschr. für angew. Math. und Mechanik*, Band 28, Heft 7/8, July–Aug. 1948, pp. 193–197.

(2) "The Scientific Papers of James Clerk Maxwell," Vol. 2, New York, Dover Publications, 1890, p. 197.

(3) S. TIMOSHENKO AND J. N. GOODIER, "Theory of Elasticity," New York, McGraw-Hill Book Co., 1951.

(4) H. M. WESTERGAARD, "Theory of Elasticity and Plasticity," Cambridge, Harvard Univ. Press, 1952, p. 123.

(5) G. MORERA, *Roma, Acc. Lincei Rend.*, Ser. 5, t. 1, 1892.

(6) H. L. LANGHAAR, "The Principle of Complementary Energy in Nonlinear Elasticity Theory," JOUR. FRANKLIN INST., Vol. 256, p. 255 (1953).

THE ACQUIREMENT OF COSMIC RAY ENERGIES BY ELECTROMAGNETIC INDUCTION IN GALAXIES *

BY

W. F. G. SWANN [1]

ABSTRACT

The paper constitutes a generalization and discussion of the principle of acceleration of charged particles to cosmic ray energies through electromagnetic induction, a principle formerly applied by the writer to a special problem.[2]

Attention is confined to problems where the magnetic field grows with time with axial symmetry, that is to problems in which the electric and magnetic fields, E and H, show no variations with the θ coordinate. It is shown that:

1. If a charged particle starts to acquire kinetic energy at $t = 0$, and if the magnetic field is zero at $t = 0$, then the particle acquires energy continually.

2. If, at $t = 0$, the magnetic field is finite, a sufficient, but by no means necessary, condition for continual gain of energy is that $|E| \geqslant |H|$ for all positions and times.

3. If, at $t = 0$, both magnetic field and particle energy are finite, then, depending upon the sign of the initial angular velocity, the particle may gain or lose energy initially. However, if $|E| \geqslant |H|$, it will eventually start to gain energy and will continue to do so.

The problem has been studied in some detail for the case where the magnetic field and kinetic energy are both zero at $t = 0$. It appears that the kinetic energy T acquired by a proton which finds itself at the cylindrical radius r, where the average z component of magnetic field is equal to $\bar{H}_z$ within that radius is such that $T > 150r\bar{H}_z$ electron volts. If $r \sim 25{,}000$ light years, and $\bar{H} \sim 7 \times 10^{-6}$ gauss, $T \sim 2.3 \times 10^{19}$ e.v.

It also appears that the particle in its motion spirals outwards, inwards, or remains on a circle, according as $\bar{H}_z > 2H_z$; $\bar{H}_z < 2H_z$; $\bar{H}_z = 2H_z$, respectively, where H_z is the average z component within the radius r at which the z component of the magnetic field has the value H_z. If, in its journey through places where $\bar{H}_z \neq 2H_z$, the particle reaches a place where $\bar{H}_z = 2H_z$, then it will asymptotically approach a stable circular orbit for that value of r.

INTRODUCTION

The present paper is an extension of certain ideas formerly suggested by the writer [2] on the acquirement of cosmic ray energies through what is frequently termed betatron activity. The considerations are confined

---

* Assisted in part by the Joint Program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission. The contents of this paper, together with many other considerations, were presented by the writer as part of the Cosmic Ray Symposium held at Durham, N. C., Nov. 30 to Dec. 2, 1953, under the auspices of the National Science Foundation. A digest of this paper has appeared as a Letter to the Editor of the *Physical Review*, issue of October 1, 1954.

[1] Director, Bartol Research Foundation of The Franklin Institute, Swarthmore, Pa.; also a Senior Adviser for The Franklin Institute Laboratories for Research and Development.

[2] W. F. G. Swann, *Phys. Rev.*, Vol. 43, p. 217 (1933); JOUR. FRANKLIN INST., Vol. 215, p. 273 (1933). A simplified version of the theory is given by the writer in the September issue of this JOURNAL, Vol. 258, p. 205 (1954).

to problems of axial symmetry about the axis of z, and to a condition in which a magnetic field changing with time has no tangential component, so that the currents producing it and the vector potential in terms of which it can be expressed are everywhere tangential to circles about the z axis. The magnetic field itself can have a radial component and a z component, however.

In what follows, r, θ, and z shall refer to the cylindrical coordinates, θ being measured in a clockwise direction as we look along the axis of z. m is the rest mass of a particle of charge e, $U(= U_\theta)$ the vector potential, r', θ', z', the radial, angular, and z components of velocity, and T the kinetic energy. If v is the velocity of the particle, β shall stand for $(1 - v^2/c^2)^{-\frac{1}{2}}$. Other symbols will be defined as they occur.

CERTAIN IMPORTANT THEOREMS

We shall first derive certain theorems governing the increase of energy of a particle in a changing magnetic field, with a view to applying them to special cases. In the latter part of this paper, one special case of wider generality than that originally studied [2] will be treated in detail to the extent of calculating a lower limit to the energy attainable in a typical case.

The Lagrangian function L for a charged particle in a field defined by a vector potential U and a scalar potential φ is

$$L = -mc^2(1 - \beta^2)^{\frac{1}{2}} + e(U \cdot v)/c - e\varphi.$$

Expressing this in polar coordinates, omitting φ, which is zero in our problem, and putting U_r and U_z zero, in accordance with the limitations of our problem, we have

$$L = -mc^2\left(1 - \frac{r^2\theta'^2}{c^2} - \frac{r'^2}{c^2} - \frac{z'^2}{c^2}\right)^{\frac{1}{2}} + \frac{e}{c}r\theta' U_\theta. \tag{1}$$

The well known dynamical theorem

$$\frac{d}{dt}(T + e\varphi) = -\frac{\partial L}{\partial t}$$

yields in our case, since $\varphi = 0$

$$\frac{dT}{dt} = -\frac{\partial L}{\partial t} = -\frac{e}{c}r\theta'\frac{\partial U_\theta}{\partial t}. \tag{2}$$

Since mc^2 is constant, we may write

$$\frac{d}{dt}(T + mc^2) = -\frac{er}{c}\theta'\frac{\partial U_\theta}{\partial t}. \tag{3}$$

Lagrange's equations, applied to the coordinate θ, yield

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \theta'}\right) - \frac{\partial L}{\partial \theta} = 0$$

so that, since L does not contain θ explicitly,

$$\frac{\partial L}{\partial \theta'} = 0$$

which, of course, expresses constancy of angular momentum, and, through (1) yields

$$mr^2\theta'(1 - \beta^2)^{-\frac{1}{2}} + erU_\theta/c = \text{const.}$$

If U_0 and r_0, θ_0 and T_0 are the values of U_θ and r, θ and T at $t = 0$, we have, since $T + mc^2 = mc^2(1 - \beta^2)^{-\frac{1}{2}}$

$$\frac{r^2\theta'}{c^2}(T + mc^2) + \frac{er}{c}U_\theta = \frac{r_0{}^2\theta_0{}'}{c^2}(T_0 + mc^2) + \frac{er_0}{c}U_0 \tag{4}$$

$$\frac{r^2\theta'}{c}(T + mc^2) - \frac{r_0{}^2\theta_0{}'}{c}(T_0 + mc^2) = e(r_0U_0 - rU_\theta). \tag{5}$$

Theorem 1. Case Where, at $t = 0$, the Magnetic Field is Zero and the Particle Has Zero Angular Velocity

In this case $\theta_0{}'$ and U_0 are zero, so that (5) becomes

$$\frac{r^2\theta'}{c}(T + mc^2) = -erU_\theta. \tag{6}$$

Multiplying the left hand sides of (3) and (6), and multiplying the right hand sides, we have

$$\frac{d}{dt}(T + mc^2)^2 = e^2\frac{\partial}{\partial t}(U_\theta{}^2). \tag{7}$$

Thus, if the *magnetic field, and so the vector potential, start from zero at $t = 0$, then, so long as the vector potential increases in absolute value with time at each point, the particle will continually increase in energy.*[3]

---

[3] As a matter of fact, it is only necessary that $U_\theta{}^2$ shall be found increasing with time at each point visited by the particle at the instant when the particle is there. The criterion cited above is, however, sufficient to guarantee increase of energy with time without appeal to the actual nature of the path described by the particle.

The result above established is contained in the writer's original paper,[2] where, however, it was subordinated to the immediate interests of the special problem then in hand. In the present paper, it will play a more vital role, and the method of its derivation is slightly different from that formerly employed in using mathematical processes perhaps more generally familiar.

Another point of interest evolves from a consideration of (5) which, in the present case, with $\theta_0' = 0$ and $U_0 = 0$ yields

$$r\theta'(T + mc^2) = -ecU_\theta$$

which, since $T + mc^2$ is positive, and U_θ is always of the same sign, shows that θ' remains of the same sign throughout. It can never be zero except at $t = 0$, and consequently, the *particle can never describe a "loop" except around the axis of symmetry*. It always describes a spiral, outwards or inwards, depending, as we shall later see, upon the nature of the magnetic field distribution. A limiting case is represented by a circular orbit, which is the case discussed in the earlier publication.[2]

Theorem 2. Case Where, at $t = 0$, the Magnetic Field is Finite, but the Particle has Zero Angular Velocity

Equation 3 holds as before, but in Eq. 5 we must retain U_0, but put $\theta_0' = 0$. On multiplying the left hand sides of (3) and (5), modified as above, and in multiplying the right hand sides of these equations, we obtain

$$\frac{1}{2}\frac{d}{dt}(T + mc^2)^2 = e^2\left(U_\theta - \frac{r_0}{r}U_0\right)\frac{\partial U_\theta}{\partial t}. \tag{8}$$

If $\dfrac{\partial U_\theta}{\partial t}$ is always positive at each point, the right hand side of (8) will be always positive if $U_\theta - r_0U_0/r$ is always positive, and hence if $rU_\theta - r_0U_0$ is always positive. This latter condition is certainly satisfied at $t = 0$, since then $U_\theta = U_0$ and $r = r_0$. Hence, it will always be satisfied if, at each subsequent position occupied by the particle,

$$\frac{d}{dt}(rU_\theta) \geqslant 0$$

that is, if

$$\frac{\partial}{\partial t}(rU_\theta) + r'\frac{\partial}{\partial r}(rU_\theta) + z'\frac{\partial}{\partial z}(rU_\theta) \geqslant 0.$$

Now it is easily seen that[4]

---

[4] Remembering that H is curl U, and that the surface integral of the normal component of curl U and so of H over a surface is equal to the line integral of U taken around the boundary, we have as follows: Taking as the surface the annular ring $2\pi rdr$, we have

$$2\pi rH_z dr = 2\pi\frac{\partial}{\partial r}(rU_\theta)dr$$

so that $rH_z = \partial(rU_\theta)/\partial r$. Again, taking as our surface the circular strip $2\pi rdz$, we have

$$2\pi rH_r dz = -\frac{\partial}{\partial z}(rU_\theta)dz$$

so that

$$-H_r = \frac{\partial}{\partial z}(U_\theta).$$

$$\frac{\partial}{\partial r}(rU_\theta) = rH_z; \quad \frac{\partial}{\partial z}(rU_\theta) = -rH_r. \tag{9}$$

Thus, our condition is

$$\frac{\partial U_\theta}{\partial t} + r'H_z - z'H_r \geqslant 0$$

$$- E_\theta - [vH]_\theta/c \geqslant 0.$$

Since U_θ increases continually with time, E_θ is continually negative, so that $-E_\theta$ is positive, so that our condition is

$$|E_\theta| - [vH]_\theta/c \geqslant 0.$$

This is certainly satisfied if

$$|E| \geqslant |H| \tag{10}$$

for all points occupied by the particle.[5]

It is thus to be observed that this condition ($|E| \geqslant |H|$) *is a sufficient condition for continual increase of energy.* It is not always a necessary condition. Thus, in the special problem already discussed in the earlier paper,[2] increase of energy with time continues after $|H|$ has become greater than $|E|$.

Again, from (5) we see that, since $\theta_0' = 0$,

$$r^2\theta'(T + mc^2) = - ce(rU_\theta - r_0U_0). \tag{11}$$

Since $rU_\theta - r_0U_0$ is always positive for the case under consideration, we see that θ' is continually negative, and so can never be zero except at $t = 0$. Thus *the particle can never describe a "loop" except around the axis of symmetry.* It always describes a spiral with a circle as a particular case.

Theorem 3. Case Where, at t = 0, the Magnetic Field is Finite, and the Particle Also Has a Finite Angular Velocity

Taking $\partial U_\theta/\partial t$ as positive, we first consider the case where θ' is positive at $t = 0$. Equation 2 shows that so long as θ' is positive, T decreases with time. Thus either θ' becomes zero before T becomes zero (because of the possible existence of a finite value for r' or z' or both) or it becomes zero by T becoming zero. In either case θ' eventually reaches the value zero. Taking a new origin of time at this instant, all the conclusions of Theorem 2 follow.[6]

---

[5] If U_θ decreases continually with time, $\partial U_\theta/\partial t$ in (8) is negative, and the argument proceeds as before, with suitable changes of sign, leading to the conclusion that (10) is a sufficient condition for continual increase of energy.

If θ' is finite at $t = 0$, we must retain θ_0' in Eq. 5, which then gives, in conjunction with (3)

$$\frac{1}{2}\frac{d}{dt}(T + mc^2)^2 = e^2\left[U_\theta - \frac{r_0}{r}U_0 - \frac{r_0^2}{rec}\theta_0'(T_0 + mc^2)\right]\frac{\partial U_\theta}{\partial t}. \quad (12)$$

This equation shows, as we have shown by other means, that at $t = 0$, (where $U_\theta r = U_0 r_0$), the energy diminishes with time if θ_0' is positive. However, it shows that if θ_0' is negative, then, regardless of its absolute value, if $U_\theta > r_0U_0/r$ for all times and positions, the energy will continually increase with time. A sufficient condition for the requirement $U_\theta > r_0U_0/r$ is, as we have seen $|E| > |H|$ for all time.

Thus again, the condition $|E| > |H|$ *is a sufficient condition for the continual increase of energy with time if* θ_0' *is negative* (*or zero*).

Further, (5) shows that if $rU_\theta > r_0U_0$, and *a fortiori*, if $|E| > |H|$, *and if* θ_0' *is negative, then* θ' *will be continually negative, and the particle can never describe a "loop" except around the origin.* It will always execute a spiral, or a circle as a particular case.

Needless to say, if we make the foregoing discussion on the basis of $\partial U_\theta/\partial t$ negative, the conclusions, with suitable and obvious changes of sign in the details remain unchanged.

ENERGIES ACQUIRED IN SOME TYPICAL CASES

So far we have confined our attention to discussing the conditions under which energy increases continually under the accelerating mechanism, and we have deduced certain characteristic features of the path under general conditions. As a rule, the calculation of the actual increase of energy over an assigned time necessitates calculation of the path of the particle. As already stated, this has been done in one special case,[2] involving a special type of field distribution. In the remainder of the present paper we shall consider a more general case.

Case Where, at t = 0, the Magnetic Field is Zero and the Particle Has Zero Velocity

Theorem 1 applies to this case, since it is a particular case of that calling only for zero angular velocity at $t = 0$. The story of the rate of increase of energy is given by (7)

$$\frac{dW^2}{dt} = \frac{d}{dt}(T + mc^2)^2 = e^2\frac{\partial U_\theta^2}{\partial t^2} = e^2\left(\frac{dU_\theta^2}{dt} - r'\frac{\partial U_\theta^2}{\partial r} - z'\frac{\partial U_\theta^2}{\partial z}\right). \quad (13)$$

---

[6] Of course, all that is involved in this whole matter is the fact that, if the particle has an initial positive value of the angular velocity θ', that angular velocity must be destroyed even at the expense of decrease in kinetic energy before the subsequent increase of kinetic energy can proceed.

In order to proceed further, it is necessary to discuss the part played by the spatial derivatives of U_θ^2, and their accompanying factors r' and z'. To this end we shall first discuss r' and z' themselves. We shall write W for $T + mc^2$.

Radial Component of the Velocity

Lagrange's equations, as applied to the r coordinate, yield, through (1)

$$\frac{d}{dt}\left(\frac{mr'}{\sqrt{1-\beta^2}}\right) - \frac{mr\theta'^2}{\sqrt{1-\beta^2}} - \frac{e}{c}\,\theta'\,\frac{\partial}{\partial r}\,(rU_\theta) = 0.$$

Since $\partial(rU_\theta)/\partial r = rH_z$, and $W = T + mc^2 = mc^2/\sqrt{1-\beta^2}$

$$\frac{1}{c^2}\frac{d}{dt}\,(Wr') - \frac{Wr^2\theta'^2}{rc^2} - \frac{e}{c}\,H_z r\theta' = 0.$$

Using (6)

$$\frac{Wr''}{c^2} + \frac{r'}{c^2}\frac{dW}{dt} - \frac{e^2U_\theta^2}{rW} + \frac{e^2H_z}{W}\,U_\theta = 0.$$

Using (7), we readily find

$$\frac{W^2r''}{c^2} + \frac{r'e^2}{c^2}\,U_\theta\,\frac{\partial}{\partial t}\,U_\theta - e^2\left(\frac{U_\theta}{r} - H_z\right)U_\theta = 0$$

$$\frac{W}{c^2}\,r'' = e^2U_\theta\left(\frac{U_\theta}{r} - H_z - \frac{r'}{c^2}\frac{\partial U_\theta}{\partial t}\right). \qquad (14)$$

This gives $r'' = 0$ at $t = 0$, since $U_\theta = 0$ at $t = 0$. However, suppose that $U_\theta/r - H_z$ is positive for all places and times. Then r' can never become negative, for so long as it is negative $-\,r'\partial U_\theta/\partial t$ is certainly positive and r'' will be positive. Hence the negative velocity will suffer reduction with time as long as it exists. The only possibility therefore is that r' will be positive. In other words, since the path cannot describe a loop except around the origin, it will spiral outwards.

Following a similar line of argument for the case where $U_\theta/r - H_z$ is negative, it is easy to draw the conclusion that r' is continually negative, and the path spirals inwards.

The z Component of the Velocity

The equations of Lagrange yield, through (1),

$$\frac{d}{dt}\,\frac{mz'}{\sqrt{1-\beta^2}} = \frac{e}{c}\,r\theta'\,\frac{\partial U_\theta}{\partial z}.$$

Using (6), and writing $W = mc^2/\sqrt{1-\beta^2}$

$$\frac{1}{c^2}\frac{d}{dt}(Wz') = -\frac{e^2U_\theta}{W}\frac{\partial U_\theta}{\partial z}$$

$$W^2z'' + z'W\frac{dW}{dt} = -c^2e^2U_\theta\frac{\partial U_\theta}{\partial z}.$$

Thus [5]

$$W^2z'' + z'W\frac{dW}{dt} = c^2e^2H_rU_\theta. \tag{15}$$

At $t = 0$, the right hand side is zero, so that if z' is zero at $t = 0$, z'' will be zero at that instant. When t is not zero, U_θ is always positive. Suppose H_r is positive, then the right hand side is positive. Thus z' can never grow to a negative value because, if it should be supposed to have such a value, z'' would be positive and the negative value would be destroyed. In other words, z' can only grow in the positive sense. Suppose, on the other hand, H_r is negative for $t > 0$. Then, by an analogous argument, we can show that z' can only grow to negative values. Thus z' is positive or negative according as H_r is positive or negative.

Return to the Main Problem

We first observe as follows: [5]

$$H_z = \frac{\partial(rU_\theta)}{\partial r} = U_\theta + r\frac{\partial U_\theta}{\partial r}. \tag{16}$$

Hence

$$\frac{\partial U_\theta}{\partial r} = -\left(\frac{U_\theta}{r} - H_z\right). \tag{17}$$

We have seen that r' is positive or negative, according as $(U_\theta/r - H_z)$ is positive or negative. Hence r' is positive or negative, according as $\partial U_\theta/\partial z$ is positive or negative. It thus results that the term $-r'\partial U_\theta^2/\partial r(= -2U_\theta r'\partial U_\theta/\partial r)$ occurring in (13) is always positive.

Again $-z'\partial U_\theta^2/\partial z(= -2U_\theta z'\partial U_\theta/\partial z)$ occurring in (13) is equal to $2U_\theta z'H_r$. Since z' is positive or negative, according as H_r is positive or negative, $2U_\theta z'H_z$, and so, $-z'\partial U_\theta^2/\partial z$, is always positive. Thus, from (13) we have

$$\frac{d}{dt}(T + mc^2)^2 > e^2\frac{dU_\theta^2}{dt}$$

and

$$(T + mc^2)^2 - mc^2 = (T^2 + 2mc^2T)^{\frac{1}{2}} > eU_\theta.$$

For convenience, but not of necessity, we can drop the term $2mc^2T$ in the case of high energies and there results

$$T > eU_\theta.$$

Here U_θ refers to the value at the position of the particle at the time when T is evaluated. If the particle is to be found on a circle of radius r at the instant concerned, we have

$$\pi r^2\bar{H}_z = 2\pi r U_\theta$$

$$U_\theta = r\bar{H}_z/2. \tag{18}$$

Thus

$$T > er\bar{H}_z/2. \tag{19}$$

Thus

$$T > 300r\bar{H}_z/2 > 150r\bar{H}_z \text{ electron volts.}$$

Thus, suppose the particle were near the boundary of the galaxy,[7] so that for Andromeda, for example, r is of the order 25,000 light years $= 25{,}000 \times 3 \times 10^{10} \times 3 \times 10^7 = 2.2 \times 10^{22}$. Thus, if $\bar{H} \sim 7 \times 10^{-6}$, we have

$$T \sim 2.2 \times 10^{22} \times 150 \times 6 \times 10^{-6} \sim 2.3 \times 10^{19} \text{ e.v.}^8$$

Difficulties Confronting the Foregoing Calculations in Their Application to Cosmic Ray Energies

While the foregoing calculation makes no use of the time during which the magnetic field has been created, but only of the final value of the field, it is natural to contemplate the growth of the field as taking place over a time of the order of the age of the universe, say 5×10^9 years. However, there is reason to believe that the life of a cosmic ray, as determined by nuclear collisions in a galaxy, is only of the order 10^8 million years, that is, 50 times shorter than the life of the universe. Thus the cosmic rays which started to acquire their energies at the "beginning of time" would no longer be with us. This particular difficulty could be overcome if we were willing to assume a periodicity in the magnetic field of the order of 4×10^8 years. It would then be possible for us to assume that the magnetic field was zero 10^8 years ago, and cosmic rays starting the acquirement of energy at that epoch could be with us now.

---

[7] This would correspond to the particle having spiralled outwards, that is, to the condition $U_\theta/r > H_z$.

[8] The value for H used is one suggested by S. Chandrasekhar and E. Fermi, *Astrophys. Jour.*, Vol. 118, pp. 113, 116 (1953).

Of course, even in the case of a magnetic field growing with time through a period of 5×10^9 years, ions produced by photoelectric action on neutral particles 10^8 years ago could have *started* their acquirement of energy at that epoch. However, such ions would be starting to acquire energy in a magnetic field which had already attained a finite value, and for this condition the requirements for the acquirement of energy are more stringent. They will be discussed in detail in a later communication.

The Role of the Space Variation of the Field

It is of interest to investigate the significance of the conditions $U_\theta/r > H_z$; $U_\theta/r < H_z$; $U_\theta/r = H_z$, which play such an important role in the foregoing development. It will be recalled that $U_\theta/r > H_z$, at a point where the particle starts to gain energy, characterizes the condition where the particle starts to spiral outwards, $U_\theta/r < H_z$ is characteristic of the case where it spirals inwards, and $U_\theta/r = H_z$ is characteristic of the case where the particle spirals neither inwards nor outwards, but describes a circle. This was the case discussed in the writer's earlier paper.[2] That the particle describes a circle for this case may also be seen directly from (14); for, if $U_\theta/r = H_z$, Eq. 14 becomes

$$\frac{W}{c^2} r'' = e^2 U_\theta = -\left(\frac{e^2 U_\theta}{c^2}\right) r' \frac{\partial U_\theta}{\partial t}.$$

If r' ever acquires a negative value, r'' will be positive so long as r' is negative, and so r' finally attains the value zero. On the other hand, if r' ever has a positive value, r'' is negative so long as r' is positive, so that r' finally attains the value zero. In other words, the orbit of the particle is an orbit with $r' = 0$. It is a circle. If, in its journey through places where $\bar{H}_z \neq 2H_z$, the particle reaches a place where $\bar{H}_z = 2H_z$, then it will asymptotically approach a stable circular orbit for that value of r.

It is interesting to observe, moreover, that if $\bar{H}_z$ is the average value of H_z within the radius r, then, by (18)

$$U_\theta = r\bar{H}_z/2.$$

Hence the conditions

$$\frac{U_\theta}{r} - H_z > 0; \quad \frac{U_\theta}{r} - H_z < 0; \quad \frac{U_\theta}{r} - H_z = 0$$

are the equivalent, respectively, of

$$\frac{\bar{H}_z}{2} > H_z; \quad \frac{\bar{H}_z}{2} < H_z; \quad \frac{\bar{H}_z}{2} = H_z.$$

Special Case Where H_z Obeys a Power Law in r

Let us consider the special case where

$$H_z = \frac{A}{r^{1+\alpha}} \tag{20}$$

where A is a function only of the time, and the absolute value of α is less than unity. We have

$$2\pi r U_\theta = A \int_0^r \frac{2\pi x dx}{x^{1+\alpha}} = \frac{2\pi A}{1-\alpha} [x^{1-\alpha}]_0^r.$$

If $\alpha < 1$, the lower limit is zero, and

$$r U_\theta = \frac{A}{1-\alpha} r^{1-\alpha} \tag{21}$$

so that, in view of (20)

$$\frac{U_\theta}{r} = \frac{H_z}{1-\alpha}.$$

Thus, U_θ/r is greater than, less than, or equal to H_z, according as α is greater than, less than, or equal to zero.[9]

---

[9] It may be observed that the singularity in H_z at $r = 0$ inherent in (20) is of no importance. As may be seen from (21), there is no singularity at $r = 0$ in $2\pi r U_\theta$, which is the flux through the radius r.

As a matter of fact, it would be easy to attach to (20) a function of r which removed the singularity at $r = 0$ and yet approached unity rapidly as r increases from zero, in such a manner as to have no material bearing upon the main features of the problem.

Film-Insulated Rectangular Wire Improves Transformers.—A new method of coating rectangular wire with film insulation has made possible several improvements in transformers, the General Electric Company's Distribution Transformer Department, Pittsfield, Mass., has announced. The improvements include an increase in transformer reliability and reductions in transformer sizes and weights, according to G-E engineers.

By a novel method, an insulation called Formex * wire film can now be applied to rectangular-shaped copper conductor. With this method the insulation is spread evenly on the edges as well as on the flat surfaces. The copper conductor is given five to eight separate coats of Formex wire film, each of which is baked.

G-E engineers reported Formex wire insulation has long been successfully used for motor coils and transformer windings of round wire.

Formex wire film strongly adheres to copper conductors and has remarkable mechanical and electrical strength, the engineers said. Even twisting, hammering the conductor out of shape, or breaking the conductor does not chip or peel the film from the turns, it was claimed.

Tests demonstrate that rectangular wire insulated with 3 mils of Formex wire film has a higher insulation strength than wire covered with 5 to 7 mils of paper or paper-and-cotton insulation. This will further improve the reliability of transformers wound with wire, G-E engineers said.

Because the space given to turn-to-turn insulation has been reduced by as much as 50 per cent with the new Formex wire, it has been possible to reduce over-all size and weight of transformers, and, at the same time, improve their electrical strength.

Use of Formex wire (together with improved silicon core steels) in a 15-kva, 7200-volt conventional transformer resulted in reducing weight from 360 to 325 pounds and the diameter from 16 to 14 inches. This is typical of the application results, G-E engineers said.

Currently Formex insulated rectangular wire is being used on sizes 3 to 15 kva inclusive, 15,000 volts and below, and on 25 kva units below 6,000 volts. Essentially, all round wires in the units are Formex insulated. The use of rectangular Formex wire will be extended to other ratings as soon as new manufacturing facilities can be installed, the engineers said.

New Flushing Oil for Vacuum Pumps.—A new oil, developed especially for flushing mechanical vacuum pumps without the risk of contamination from detergents or volatile additives, is announced by Central Scientific Company, Chicago, Ill.

The company explains that detergents or volatile additives found in automobile lubricating oils can readily contaminate a high vacuum system. Because of the characteristics of high vapor pressure, removal of additive or solvent traces may require hours of pumping or heating. In some cases, the contamination may be nearly impossible to remove.

The new Cenco Hyvac Flushing Oil eliminates contamination hazards and is ideal for removing old oil, sediment and dirt which may accumulate in the internal mechanism of the pump. One or two flushings, followed by a refill with Cenco Hyvac Oil, is all that is needed to prepare a pump for immediate use.

Cenco Hyvac Flushing Oil comes in quart bottles or gallon cans and is intended for flushing only and not for high vacuum pumping.

---

* Registered trade mark of the General Electric Company for wire and cable.

ELECTROMAGNETISM, OLD AND NEW

BY

F. W. WARBURTON[1]

In recent articles, Moon and Spencer (1, 2)[2] claim to have developed the only possible expression for force between moving charges consistent with the four statements of Ampère expressing the result of his experiments, and consistent with their own postulates A, B, C, D, E. They designate the mutual force of moving charges as the Ampère force and add terms which they call the Weber force, a function of acceleration of the charge q_1, and the Maxwell force, a function of growth and decay of charge. That their result is incomplete is seen at once when one notes that the Weber and Riemann forces and the more general reciprocal formula (3)

$$\mathbf{F} = e_1e_2\left[\frac{\mathbf{r}}{r^3} - \tfrac{3}{2}(1 - 2b)\frac{\mathbf{r}(\mathbf{u}\cdot\mathbf{r})^2}{c^2r^5} + (1 - b)\frac{\mathbf{r}u^2}{c^2r^3} - 2b\frac{\mathbf{u}(\mathbf{u}\cdot\mathbf{r})}{c^2r^3} + (1 - 2b)\frac{\mathbf{r}(\mathbf{g}\cdot\mathbf{r})}{c^2r^3} + 2b\frac{\mathbf{g}}{c^2r} + \cdots\right], \quad (1)$$

not only are known to satisfy Ampère's experiments but also that these all satisfy postulates A, B, C, D, E. This is the case both when $\mathbf{g} = 0$, the condition of zero acceleration specified by Moon and Spencer when discussing the Ampère force, and also when the postulates are extended to include relative acceleration. Their result for charges having relative velocity, Eq. 18 (1, p. 313), is simply Weber's force equation (which is expressed in Eq. 1 by setting $b = 0$) under this condition of constant velocity, $\mathbf{g} = 0$. Like the expression given by Ampère for current elements, it is unnecessarily limited (4) by having only the direction of r. It would seem better to have acknowledged Weber's contribution at this point, than to have designated these forces of relative motion of individual charges as "Ampère" forces. The recognition of the Ampère force for current elements is already confused in the literature by attributing to Ampère another form for current elements, $d^2\mathbf{F} = id\mathbf{l} \times i'd\mathbf{l}' \times \mathbf{r}/r^3$, which Ampère considered invalid for elements and valid only for the closed circuit.

In terms of the angles used by Moon and Spencer, θ_1 and θ_2, which the current elements make with the distance r between e_1 and e_2, and the angle η between their corresponding planes, Eq. 1 above yields, with axes oriented so that $\mathbf{i}$ is a unit vector in the r-direction, and with $\mathbf{u} = 0$,

---

[1] Department of Physics, University of Redlands, Redlands, Calif.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

$\mathbf{v}$, $-\mathbf{v}'$, $\mathbf{v} - \mathbf{v}'$ and $\mathbf{g} = 0, \mathbf{f}, -\mathbf{f}', \mathbf{f} - \mathbf{f}'$ successively,

$$\begin{aligned}\mathbf{F} = (e_1v_1e_2v_2/c^2r^2)[\mathbf{i}\{\cos\theta_1\cos\theta_2 - 2(1-b)\sin\theta_1\sin\theta_2\cos\eta\} \\ + \mathbf{j}\{2b(\cos\theta_1\sin\theta_2 + \sin\theta_1\cos\theta_2\cos\eta\} \\ + \mathbf{k}\{2b\sin\theta_1\cos\theta_2\sin\eta\}]. \quad (2)\end{aligned}$$

Moon and Spencer evidently fail to note that in their Eq. 10, $f_y(v, \theta)$ is not necessarily equal to $f_r(v, \theta)$ or to $f_z(v, \theta)$, so that substitution of solutions for f_r in f_y, f_z is thus not valid; nor do they note that $n = 1$, $q = 1$ in $f_y(v, \theta)$ with $f_z(v, \theta) = 0$ provides a satisfactory solution of both their $\mathbf{a}_y$ and $\mathbf{a}_z$ terms, conforming to Eq. 2 and Eq. 1.

Assigning the names Weber and Maxwell to specific forces with which the writings of these eminent men do not agree would engender further confusion. The second and third sentences of the first paragraph above illustrate this confusion, as the Weber force mentioned in the third sentence is that developed by Weber. The so-called "Weber" force proposed by Moon and Spencer may be obtained from Eq. 1 with $b = \frac{1}{2}$, and evidently satisfies postulates A, B, C, D, E if postulates B and E be extended to include the acceleration term proportional to the inverse first power of r.

The so-called "Maxwell" force is superfluous. The charge as defined changes its magnitude with time. This force thus does not conform to the criterion of a basic force formula for electrodynamics consisting of forces of individual whole electrons or protons, each acting on another electron or proton with forces additive, which is the essence of postulate E. This proposed "Maxwell" force is a function of gradual birth and death of macroscopic charge only, that is the successive appearance and disappearance of many discrete electrons, without regard to where they came from. And it does not apply to a single electron, for example, which arrives on a condenser plate along some path with velocity and acceleration, but is not born or created, nor buried or annihilated there. Lorenz (5) developed his theory of propagated potentials in terms of a macroscopic charge density $\epsilon(t - r/c)$ which is a function of time. Riemann however, in the preceding paper (6), which had been laid before the Royal Society of Sciences at Göttingen on 10 February 1858, expressed propagation of potentials of fixed discrete charged particles ϵ, ϵ' in terms of the variable distance r between them, as a function of time, $r(t - \alpha/c, t)$.

Inasmuch as propagation of potentials is assumed with the reciprocal forces of Eq. 1 above, and inasmuch as assumption of propagation of the classical potentials replaces the fictitious Maxwell displacement current (7) and provides the same set of Maxwell equations, one concludes that retarded solutions of macroscopic propagation equations satisfying macroscopic boundary conditions are compatible with the reciprocal electrodynamics of Eq. 1. These macroscopic solutions of practical problems set up in terms of birth of macroscopic charge, without regard to where the electrons come from or who their father was, have retarded forms for the gradient of potential, $\mathbf{r}_0 \left(\frac{1}{r} - \frac{i\omega}{v}\right) \frac{e^{i\omega r/v}}{r}$, or Eq. 18 (2, p. 377). These macroscopic solutions neither reveal how the electrons arrive nor deny that the reciprocal force, Eq. 1, is adequate. It is difficult to see how the partial gradient of the retarded potential, $\partial(Q(t - r/c)/r)/\partial r$, Eq. 25 (2, p. 381), is closely allied with or replaces Maxwell's displacement current except insofar as it provides a result comparable with solutions of a propagation equation for a particular macroscopic case.

One of the difficulties with the use of the Maxwell Equations, which were developed on macroscopic (and loose) concepts, has been implicit faith in their exactness for microscopic details. Invention of a microscopic force to fill an exclusively macroscopic condition is an error in the opposite direction. It may be that such an improper condition imposed on the expression for reciprocal force, Eq. 1 above, led Moon and Spencer to their unsupported claim (8) that its application to a simple antenna problem yields incorrect results.

The potential energy of two charges e_1 and e_2 includes the radiation picked up by e_2. Whittaker (9) has shown that the Weber force formula is obtained by using a generalized potential energy function $V = (ee'/r)(1 + \dot{r}^2/2c^2 + \cdots)$ in the Lagrangian equations of motion. More recent work (unpublished) shows that an equivalent method (10) which may be used in obtaining Eq. 1 by taking the average negative gradient of potential energy, is quite general. The types of forces found in nature—gravitational, magnetic and electrostatic—are not functions of position alone, but each has been found to involve relative velocity and acceleration—gravitational force in the advance of the perihelion of the planets, and the magnetic force of moving charges in the variation of electrostatic force. The complete potential energy, defined as the work which one body or charge e_1 can do on another body or charge e_2, if and when its influence reaches e_2, is then not a function of position alone but includes the relative velocity. A small bit of radiation picked up by e_2, in the radiation field $\mathbf{E} = -\partial\mathbf{A}/c\partial t$ of an oscillating charge e_1, is like the small bit of energy of the induced electric field $\mathbf{E} = -\partial\mathbf{A}/c\partial t$ of the accelerated electron of the primary circuit picked up by e_2 in the secondary circuit of a power transformer. Both fields $\mathbf{E}$ vary as $1/r$ and each charge e_2 lets the remainder of the energy go by to be picked up by other e_2's. In the power transformer the time of propagation is negligible. The same type of energy function is used in both cases. It is thus incorrect to conclude that the Weber force "must necessarily fail when there is radiation."

REFERENCES

(1) P. MOON AND D. E. SPENCER, JOUR. FRANKLIN INST., Vol. 257, p. 305 (1954).
(2) P. MOON AND D. E. SPENCER, JOUR. FRANKLIN INST., Vol. 257, p. 369 (1954).
(3) F. W. WARBURTON, *Phys. Rev.*, Vol. 69, p. 40 (1946).
(4) E. T. WHITTAKER, "Theories of the Aether and Electricity," New York, Longmans, Green, 1910, p. 90.
(5) M. L. LORENZ, *Ann. d. Physik und Chemie*, Vol. 131, p. 243 (1867); *Phil. Mag.*, Vol. 34, p. 287 (1867).
(6) BERNARD RIEMANN, *Ann. d. Physik und Chemie*, Vol. 131, p. 237 (1867); *Phil. Mag.*, Vol. 34, p. 368 (1867).
(7) F. W. WARBURTON, *Am. J. Phys.*, Vol. 22, p. 299 (1954).
(8) P. MOON AND D. E. SPENCER, *Am. J. Phys.*, Vol. 22, p. 120 (1954).
(9) E. T. WHITTAKER, "Analytical Dynamics," Third edition, Cambridge University Press, 1927, p. 44.
(10) F. W. WARBURTON, *Phys. Rev.*, Vol. 74, p. 1562A (1948).

ELECTROMAGNETISM, OLD AND NEW: A REPLY

BY

PARRY MOON[1] AND DOMINA EBERLE SPENCER[2]

We are indebted to Dr. Warburton for his thought-provoking comments on electrodynamics. The subject is of such interest and has such wide ramifications that a thorough discussion is desirable. The principal criticisms of our formulation, as we understand them, are as follows:

(I) The Warburton equation is correct; our equation is incorrect.
(II) Our proof of the uniqueness of the equation for the Ampère force is invalid.
(III) The Maxwell force is superfluous.

We shall consider these criticisms in the above order.

(I) In the paper, "A New Electrodynamics" (**2**),[3] we give a single equation (25, p. 381) which we believe contains within itself the essentials of electrodynamics. In other words, the four equations of Maxwell and the multitude of electrical-engineering equations should be replaceable by this single equation. The idea of one equation for electrodynamics is not new (**8**, p. 123): such equations were developed previously by Gauss (1835), by Ritz (1908), and by Warburton (1946). The criterion for acceptance or rejection of any of these equations is a pragmatic one: if a single example can be exhibited in which the proposed formula does not agree with experiment, the equation should be rejected.

---

[1] Department of Electrical Engineering, Massachusetts Institute of Technology, Cambridge, Mass.

[2] Department of Mathematics, University of Connecticut, Storrs, Conn.

[3] The boldface numbers in parentheses refer to the references appended to the preceding paper.

As a simple example, we considered (2) the elementary dipole antenna with sinusoidal a-c. excitation. The details of the application of our formula are given on pp. 379 and 380. The result is identical with that obtained from Maxwell's equations:

$$\left.\begin{aligned} F_r{}^*/Q_2 &= \frac{[I^*]l\cos\theta}{2\pi\epsilon c}\left[\frac{1}{r^2}-\frac{ic}{\omega r^3}\right], \\ F_\theta{}^*/Q_2 &= \frac{[I^*]l\sin\theta}{4\pi\epsilon c}\left[\frac{1}{r^2}-\frac{ic}{\omega r^3}+\frac{i\omega}{cr}\right]. \end{aligned}\right\}$$

Since our equation gives the accepted result for the infinitesimal antenna, it also gives the accepted result for the long antenna, the wave antenna, and for antenna arrays, because these equations are obtained by integrating the equation for the infinitesimal dipole.

Now take the Warburton equation (Eq. 1, p. 395 of this issue of the JOURNAL). To agree with Postulates A to E, the equation must have $b = 0$. Straightforward application of the Warburton equation to the dipole yields

$$\left.\begin{aligned} F_r{}^*/Q_2 &= \frac{[I^*]l\cos\theta}{4\pi\epsilon c}\left[\frac{1}{r^2}-i\left(\frac{\omega}{cr}+\frac{2c}{\omega r^3}\right)\right], \\ F_\theta{}^*/Q_2 &= -\frac{i[I^*]l\sin\theta}{4\pi\epsilon\omega r^3}. \end{aligned}\right\}$$

Not only is the magnitude incorrect, but even the direction of the field is wrong. At large distances from the antenna ($r \gg l$), the Warburton equation predicts that the field is radial, while experiment shows that **E** is in the θ-direction as given by our equation.

In view of this result, speculations about the correctness of Eq. 1, based on retarded potentials or the Lagrangian or energy distribution in space or possible identity of electricity and gravitation, seem rather academic. Unless Dr. Warburton can show how his equation can be made to give the correct result for the antenna problem, we must regard his equation as invalid.

(II) The next question deals with the Ampère force—the force that occurs because of relative motion of unaccelerated charges. Warburton claims that $f_y = v^2 \sin\theta\cos\theta$, $f_z = 0$ "provides a satisfactory solution for . . . $\mathbf{a}_y$ and $\mathbf{a}_z$ terms." This is not true, as can be seen by substitution into our Eq. 10 (1, p. 310). Moreover, we are quite aware that f_x, f_y, and f_z are not necessarily of the same form. Equation 12 is a generic form applying to any of the three functions. All combinations were tried, and all but one were found wanting.

We continue to believe, therefore, that *there is only one equation* that satisfies Postulates A to E, and this equation is the one derived by Gauss and used in our general equation. It is not the equation of Weber, it is not the equation of Riemann, and it is not the equation of Grassmann. The sooner these equations are forgotten, the better.

(III) The third question deals with the Maxwell force. Warburton's claim, "the so-called Maxwell force is superfluous," is contradicted by fact. With this term, Eq. 25 gives the correct result for the dipole antenna; without this term, it gives an incorrect result.

Confusion seems to exist also as to the meaning of our "charged particles." These particles are *not* isolated electrons or protons. To speak of the position or velocity of a single electron is, of course, meaningless. In this respect, all inter-particle-force formulas are macroscopic rather than microscopic. What one is really interested in is an elementary volume, very small but containing nevertheless an immense number of electrons. Mathematically, one allows his infinitesimal volume to approach zero, but physically he considers that the shrinking stops when the volume still contains perhaps 10^{10} electrons. If this "infinitesimal volume" or "particle" is perfectly insulated, Q is constant and the Maxwell force is zero. But if the volume is part of a conductor, then Q is generally a function of both time and position, $\partial Q/\partial t \neq 0$, and a Maxwell force exists.

Finally, as to notation, we found it difficult to talk about the various components of force without having names for them. The suggested names (Coulomb, Ampère, Weber, and Maxwell) are somewhat arbitrary—as are all names—but they should not be confusing. Today, practically no one has heard of the Weber equation. It was properly buried almost a century ago; and if Dr. Warburton does not persist in digging it up, there will be no trouble.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS*

SEAL-OFF VALVE FOR VACUUM SYSTEMS

A seal-off valve possessing several advantages for high-vacuum work has been devised by R. J. Richards of the National Bureau of Standards. The valve is designed so that its handle and stem can be readily removed and used with any number of seal-off seats, leaving only the closed valve seat on the evacuated apparatus. In this way the vacuum system is protected from the atmosphere at all times yet the bulkiness of a complete permanent valve is avoided. The device is also well adapted to use as a pressure seal or as a combination safety valve and seal-off valve. It is now being used for high-vacuum applications in work sponsored by the Atomic Energy Commission at the NBS–AEC Cryogenic Engineering Laboratory.

Vacuum systems are commonly sealed off either by installing a complete valve or by pinching off annealed metal tubing, with or without the use of solder. Both types of tubing seals require the soldering or welding of a new piece of tubing into the system each time it is evacuated. This not only results in exposure of the vacuum system to the atmosphere but often presents difficult problems of manipulation. A complete valve is objectionable because it is frequently an awkward protuberance on a small piece of apparatus.

The NBS valve provides a permanent pumping tap that can be used many times without soldering or welding. Principal parts are a valve handle and stem, the valve body and seat, and a removable bonnet enclosing the seat assembly and adjacent end of the stem. The movable seat contains six ports through which the vacuum system is pumped out. Attached to this end of the stem is a narrow bar which contains two small pins equidistant from the axis of the stem. When the desired vacuum has been obtained, the pins are made to engage two of the ports in the seat, and the handle is turned until the valve is seated. The screws of the bonnet are then loosened, and all the parts except the movable seat assembly are removed as a unit. A rubber or plastic cap may be used to protect the remaining parts from dirt. When the static vacuum space requires pumping again, the removable parts are replaced, the bonnet screws are tightened, and the valve handle is turned to open the valve.

Because of the very short ports in the valve, the valve body can be made very small without sacrificing pumping speed. The parts can be made of any suitable material, but a neoprene seat is recommended for use on high-vacuum systems.

---

* Communicated by the Director.

CALIBRATION OF VIBRATION PICKUPS AND ACCELEROMETERS

In recent years the widespread use of vibration pickups has introduced a serious calibration problem. The increase in the number of pickups has emphasized the need for rapid, reliable means of calibration over wide ranges of frequency, amplitude, and acceleration. Several methods which are proving useful in the accurate calibration of vibration pickups and accelerometers have recently been developed at the National Bureau of Standards in connection with a program of basic instrumentation research sponsored by the Office of Naval Research, the Air Research and Development Command, and the Atomic Energy Commission. In one method, a "chatter" accelerometer determines very accurately the peak value of a sinusoidal acceleration, when this value is slightly in excess of one g. The device is useful in setting the acceleration level of vibration generators which are used in the calibration of vibration pickups. In another method, a steel bar is mounted on a conventional vibration generator. A vibration pickup is attached to one end of the bar, and the system is driven at its resonant frequency. Use of the resonance effect achieves values of acceleration up to $1090g$—much higher than would be attainable if the pickup, without the steel bar, were mounted directly on the same type of shaker.

The Chatter Accelerometer

The "chatter" principle was first described by Sir William Bragg in 1919. The accelerometer based on this principle is a device in which a loose mass on a vertically vibrating surface bounces or "chatters" when the peak acceleration of the sinusoidal motion exceeds $\pm 1g$. Using an improved accelerometer developed by C. W. Kissinger of NBS, the acceleration of the shaker table to which the device is attached, as well as the chatter of the loose mass, is converted into an electrical signal which is displayed on the screen of an oscilloscope. The mathematical relationship was also derived between (1) the amount by which the peak value of the sinusoidal acceleration exceeds $1g$ and (2) the point in the cycle at which the first bounce occurs. If this mathematical relationship is applied to the accelerometer output as observed on the oscilloscope screen, acceleration levels from about $1.01g$ to $1.04g$ may be set with an accuracy of approximately 0.2 per cent at frequencies up to 60 cps, and with somewhat decreased accuracy up to about 150 cps. The accuracy of calibration was estimated by comparison with an optical calibration in which the peak-to-peak displacement of the chatter accelerometer was measured with a micrometer microscope. Stroboscopic illumination makes the accelerometer appear to be nearly motionless for ease of measuring displacement.

In the Bureau's chatter accelerometer, the loose mass can be clamped by a screw and prevented from chattering. The accelerometer can then be used as a secondary standard for accelerations in excess of 1g at frequencies up to 1200 cps.

Steady-State Calibration

The method for steady-state calibration at high values of sinusoidal acceleration was developed by T. A. Perls and C. W. Kissinger of the NBS staff. In this method, a steel bar is clamped to the head of an electrodynamic shaker at its center point, so that both ends of the bar are free. The accelerometer being tested is clamped back-to-back with an NBS–33–14 secondary-standard accelerometer at one end of the bar, and a balancing weight is fastened to the other end, if needed. The whole system is driven at its resonant frequency by a 70-watt amplifier. Steady-state accelerations have been measured with this system up to 1090g, zero to peak, at 874 cps. When the resonant bar was additionally loaded with a 40-gram test accelerometer, accelerations up to 665g were attained. In comparison, if the steel bar had not been used and the accelerometer had been mounted directly on the shaker head, the maximum attainable acceleration would have been only 50g.

A series of tests was conducted at the Bureau in which the steel bars used as part of the resonant system were first tested in the annealed state, and then heat-treated and again tested. Results indicate that the heat treatment increases the maximum steady-state acceleration two-fold. In most cases the vibrational amplitude did not remain proportional to the shaker excitation current over the full available power range. Thus, the elastic properties of the bars used in the tests limit the maximum steady-state vibratory amplitudes attainable by this method.

In this method, the armature of the electrodynamic shaker is part of the resonating system, and its amplitude of motion at resonance is proportional to the amplitude at the ends of the bar. This mode of operation makes it possible to determine that the output of an NBS–33–14 accelerometer from 25 to 1090g at 874 cps. is proportional, within 2 per cent, to the output of a velocity pickup built into the armature of the shaker.

THE FRANKLIN INSTITUTE

MINUTES OF THE MEDAL DAY MEETING

Wednesday, October 20, 1954

The annual Medal Day Ceremonies were held on Wednesday, October 20, 1954. Approximately 390 persons attended the reception and dinner in Franklin Hall. Guy Marriner, Director of Music, played the National Anthem. The invocation was delivered by the Reverend Frederick R. Griffin, D.D., Minister Emeritus of the First Unitarian Church, Philadelphia, Pennsylvania. S. Wyman Rolph, President, thanked the Hostess Committee for the floral decorations and for their assistance during the reception. The dinner was concluded with a toast to Benjamin Franklin proposed by the President.

Guests adjourned to the Lecture Hall, where the President called the meeting to order at 8:30 P.M. The Stated Monthly Meeting was held in conjunction with the Medal Day Exercises. Minutes of the Stated Monthly Meeting in May were approved as printed in the June issue of the JOURNAL.

President Rolph reported that the Board of Managers, at its meeting on September 15th, had nominated as prescribed in the By-Laws, Walter A. R. Pertuch, Librarian, for Honorary Membership in recognition of his loyalty and conscientious service during his fifty years of employment. Institute members present voted unanimously in favor of his election, and the President declared him an Honorary Member.

Professor William S. Pardoe, Chairman of the Committee on Science and the Arts, introduced thirty-three former Medalists and representatives of ten former Company Medalists who were present.

Henry B. Allen, Executive Vice-President and Secretary, reported on "The Institute in 1954"—a comparison of the aims of our founders with our activities and objectives today.

Following Dr. Allen's report, presentation ceremonies were opened by a short message from the President. The sponsor of each Medalist read the citation accompanying the award and introduced the Medalist. At the conclusion of the awards, C. E. Kenneth Mees, Franklin Medalist, gave an interesting talk on "Fifty Years of Photographic Research."

The meeting adjourned at 10:05 P.M.

(The full proceedings of Medal Day will be published in the December issue of the JOURNAL.)

HENRY B. ALLEN
Secretary

PROGRAM

Reception for Medalists	THE HOSTESS COMMITTEE
National Anthem	GUY MARRINER
Invocation	THE REVEREND FREDERICK R. GRIFFIN, D.D.
Dinner	MEDALISTS, OFFICIALS AND GUESTS
Toast to Benjamin Franklin	S. WYMAN ROLPH, *President*

ADJOURNMENT TO LECTURE HALL

Greetings	THE PRESIDENT
Stated Meeting of the Institute	THE PRESIDENT PRESIDING
Introduction of Former Medalists	WILLIAM S. PARDOE, *Chairman, The Committee on Science and Arts*
The Institute in 1954	HENRY B. ALLEN, *Executive Vice-President*
Presentation of Awards	THE PRESIDENT
"Fifty Years of Photographic Research"	C. E. KENNETH MEES

MEDALISTS

To be Awarded Levy Medals (1923)

ALFRED C. BLASCHKE
Douglas Aircraft Company
Santa Monica, California

CRAIG L. TAYLOR
Professor of Engineering
University of California
Los Angeles, California

To be Awarded a Henderson Medal (1924)

C. LEVON EKSERGIAN
Executive Engineer and
Assistant to Vice President
The Budd Company
Philadelphia, Pennsylvania

To be Awarded Wetherill Medals (1925)

WILLIAM DONALD BUCKINGHAM
Assistant Electronics Research Engineer
The Western Union Telegraph Company
Electronics Research Laboratory
Water Mill, Long Island, New York

CLARENCE NICHOLS HICKMAN
Research Consultant
Jackson Heights, New York

EDWIN T. LORIG
Chief Development Engineer
United States Steel Corporation
Pittsburgh, Pennsylvania

To be Awarded Brown Medals (1938)

EDMUND H. GERMER
Consulting Physicist
Irvington, New Jersey

HUMBOLDT W. LEVERENZ
Director, Physical and Chemical Laboratory
Radio Corporation of America
RCA Laboratories
Princeton, New Jersey

HANS J. SPANNER
Consultant in Electrical Engineering
Zürich, Switzerland

To be Awarded a CLAMER MEDAL (1943)

WILLIAM JUSTIN KROLL
Consulting Metallurgist
Corvallis, Oregon

To be Awarded a Ballantine Medal (1946)

KENNETH ALVA NORTON
Chief of Radio Propagation Engineering Division
National Bureau of Standards
Boulder, Colorado

To be Awarded the Franklin Medal (1914)

C. E. KENNETH MEES
Vice President in Charge of Research
Eastman Kodak Company
Rochester, New York

LIBRARY

The Committee on Library desires to add to the collections any technical works that members may wish to contribute. Contributions will be gratefully acknowledged and placed in the Library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat service. Photostat prints of any material in the collections can be supplied on request.

The Library and reading room are open from 9 A. M. until 5 P. M. on Mondays, Tuesdays, Thursdays, Fridays and Saturdays; 2 P. M. until 10 P. M. on Wednesdays.

RECENT ADDITIONS

AERONAUTICS

EAMES, JAMES P. Turbine and Jet Propelled Aircraft Power Plants. 1954.

BIBLIOGRAPHY

STRATICO, S. Bibliografia di Marina. 1823.

BIOLOGY

SCHWIEGK, HERBERT. Künstliche Radioaktive Isotope in Physiologie Diagnostik und Therapie. 1953.

CHEMISTRY AND CHEMICAL TECHNOLOGY

ARIES, R. S. & ASSOCIATES. Marketing Research in the Chemical Industry. 1954.
COULSON, J. M. AND RICHARDSON, J. F. Chemical Engineering. Vol. 1. 1954.
FARKAS, ADALBERT AND WIGNER, EUGENE, ED. L. Farkas Memorial Volume. 1952.
FIERZ-DAVID, HANS EDWARD AND BLANGEY, LOUIS. Farbenchemie. 8 Aufl. 1952.
HADERT, HANS. Neuesrezeptbuch für die Farben und Lackindustrie. Bd. 1. 1952.
HARTMANN, HERMANN. Theorie der Chemischen Bindung. 1954.
UHLENBROCK, JAN HENDRIK. Synthese van Phosphalidezuren. 1953.
VANDANGE, C. Matériel et Genie Chimiques. 1953.
VAN DIJK, CHRISTIAAN PIETER. Bereiding, Smaak en Anaesthelische Werking van eenige Alkoxy-Amino-Nitrobenzenen. 1946.
WINCHELL, ALEXANDER N. The Optical Properties of Organic Compounds. Ed. 2. 1954.
WOLFF, GEORGES AND WOLFF, JEAN PIERRE. Méthodes d'Analyse et de Contrôle Industriel des Matiéres Grasses. 1953.

ELECTRICITY AND ELECTRICAL ENGINEERING

BÖDEFELD, THEODOR AND SEQUENZ, HEINRICH. Elektrische Maschinen. 4 Aufl. 1952.
CARTER, G. W. The Electromagnetic Field in its Engineering Aspects. 1954.
NECKENBERGER, ERNST. Elektrische Shallübertragung Grundlagen Elektroakustischer Energie-Wandlung und-Verstärkung. 1 Aufl. 1953.
ROTHE, HORST. Elektronenröhren-Physik in Einzelberichten. 1953.
STRUTT, M. J. O. Transistoren. 1954.
WARE, LAWRENCE ALBERT AND TOWN, GEORGE R. Electrical Transients. 1954.

ENGINEERING

MURPHY, LOUIS J. Plant Engineer's Easy Problem Solver. 1954.
OSGOOD, WILLIAM R., ED. Residual Stresses in Metals. 1954.

MANUFACTURE

WERNER, ARTHUR WILLIAM. The Manufacture of Fibre Boxes. Ed. 3. 1954.

MATHEMATICS

BLANC-LAPIERRE, ANDRÉ AND FORTET, ROBERT. Théorie des Fonctions Aléatoires Applications. 1953.
FISCHER, BERNHARD. Elements of Mathematics for Radio, Television, and Electronics. 1954.

Libri, Guillaume. Histoire des Sciences Mathematiques en Italie, Depuis la Renaissance des Lettres. T. 1–4. 1838–1841.

Reichenbach, Hans. Nomological Statements and Admissible Operations. 1954.

Soroka, Walter W. Analog Methods in Computation and Simulation. 1954.

Vinogradov, I. M. The Method of Trigonometrical Sums in the Theory of Numbers. 1954.

MECHANICAL ENGINEERING

Henriot, G. Traité Théorique et Pratique des Engrenages. Ed. 2. T. 1. 1954.

Kaufmann, Walther. Vorträge über Mechanik als Grundlage für das Bau-und Maschinenwesen. 8 Aufl. T. 1. 1927.

MEDICINE

Van Went, Johanna M. Ultrasonic and Ultrashort Waves in Medicine. 1954.

METALLURGY

Hall, E. O. Twinning and Diffusionless Transformations in Metals. 1954.

PHOTOGRAPHY

Schenck, Hilbert van Nydeck and Kendall, Henry. Underwater Photography. 1954.

PHYSICS

Beeler, Nelson Frederick and Branley, Franklyn M. Experiments with Atomics. 1954.

Beranek, Leo Leroy. Acoustics. 1954.

Boyle, Robert. New Experiments Physico-Mechanicall. 1660.

Flügge, Siegfried. Rachenmethoden der Quanten Theorie. 2 Aufl. T. 1. 1952.

Frankl, F. I. and Karpovich, E. A. Gas Dynamics of Thin Bodies. 1953.

Fünfer, Ewald and Neuert, Hugo. Zählrohre und Szintillationszähler Messmethoden für die Strahlung der Künstlich und Natürlich Radioaktiven Substanzen. 1954.

Heisenberg, Werner, ed. Kosmische Strahlung. 1953.

King, Horace Williams. Handbook of Hydraulics. Ed. 4. 1954.

Knudsen, James G. and Katz, Donald L. Fluid Dynamics and Heat Transfer. 1954.

Martin, Charles Noël. Numerical Tables of Nuclear Physics. 1954.

Pascal, Blaise. Traite de l'Equilibre des Liquers et de la Pesanteur de la Masse de l'Air. 1663.

SCIENCE

Bowen, Harold G. and Kettering, Charles. A Short History of Technology. 1954.

SUGAR

The Importance of Jamaica to Great Britain. 1740.

JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in this Journal within the next few months:

Wathen, Robert L.: The Traveling Wave Tube—A Record of Its Early History.

Moon, Parry and Domina Eberle Spencer: Photometrics in Astronomy.

Pomerantz, Martin A.: Primary Alpha Particles in the Cosmic Radiation Near the Geomagnetic Equator.

Plyler, Earle K.: Infrared Measurements, Past and Present.

Mishkin, E.: Disturbances in the Induction Machine Due to Broken Squirrel-Cage Rings.

Swann, W. F. G.: Shear Modulus and Viscosity Relations in Plastic Materials.

Kaiser, H. F.: Microtron Resonators.

Forscher, Frederick: Analysis of Continuity of One Phase in a Powder Mixture of Two Phases.

MEMBERSHIP

ACTIVE MEMBERS ELECTED AT THE MEETING OF THE BOARD OF MANAGERS, OCTOBER 20, 1954

ACTIVE FAMILY

J. L. Behmer
Charles A. Cary
Philip M. Lessig, M.D.
Edward D. Meanor
Martin J. Newburger
William C. Rowland

ACTIVE

John William Allen
George H. Austin, M.D.
Harry H. Boardman
Attilio Castiglioni
James B. Castner
Wendell N. Currier
Rudolf De Jong
Robert H. Elliott, Jr.
Kenneth E. Ford
Morris Freed
Walter Gazdik
Stanley K. Herbets, M.D.
Bernard Hoffman
John Johnston, Jr.
Leo Kagan
Jack Karnell
William H. Knowles
Sidney Lipschutz
Isidor Moskowitz
LeRoy C. Oberholtzer
Lewis T. Riddell
Herman L. Ruben
Charles A. Sanford
Tom Tang
Edwin Wolf, 2nd

ACTIVE NON-RESIDENT

William A. Smiley

NECROLOGY

Alexander Klein '46
Harvey Lechner '45
Jane Graham Mason '34
R. Sanford Saltus '37
Clarence G. Shenton '43

TO MEMBERS AND THOSE WHO KNOW THE FRANKLIN INSTITUTE

All of us associated with The Franklin Institute are gratified by the support given by its benefactors in the past. Without their generosity our privately endowed Institute would have been unable to serve industry, the professions, and the public as it has and should.

There is still a continuing and pressing need of contributions and bequests—both for general and specific endowment, and of income to be judiciously used. Your response will be most earnestly appreciated.

Checks should be made payable to The Franklin Institute, and sent to The Franklin Institute, Benjamin Franklin Parkway at 20th Street, Philadelphia 3, Pennsylvania.

Where property is transferred, title should be in the name of The Franklin Institute of the State of Pennsylvania for the Promotion of the Mechanic Arts.

We shall be glad to supply any additional information regarding gifts and memorials. Please write to the above address, attention of the Endowment Department.

NOTES FROM THE FRANKLIN INSTITUTE MUSEUM

A. C. CARLTON, DIRECTOR

BOUNCING BALLS

If ever an ambitious writer should undertake to record the story of man's laziness he will have to devote a chapter to ball bearings. While it would scarcely be an exaggeration to say that non-friction bearings made the expansion of the machine possible, they were invented to conserve human energy. From the time when man began to keep a record of his doings he seems to have known that round stones and rollers would reduce the effort required to move heavy objects.

It was the invention of the wheel that brought plain bearings into use, and when wheels were made of metals, lubricants had to be introduced to make the plain bearings efficient. With the arrival of the steam engine friction graduated from the level of a minor annoyance to that of a major obstacle. Anti-friction bearings became a pressing matter. Yet, neither steam, nor electrical, nor automotive engineering produced the ubiquitous ball bearing. It came when the bicyclist was relieved of the savage effort to pedal his machine up a hill. Since 1816, when Baron von Drais made his first velocipede, there have been over 5000 patents relating to bicycles, but none have been so influential as the pneumatic tire and ball bearings.

At the time when cycling was at the peak of its popularity, seven companies in this country were engaged in the making of ball bearings, but after the pastime began to languish at the turn of the century, the infant bearings industry almost suffered extinction. The basic patent for roughing out the blanks on grinders was granted an employee of the Waltham Emery Wheel Company in 1887. Later, this patent passed to the Cleveland Machine Screw Company and, in 1890, this firm made a number of machines for a bearing company in Germany, who wished to meet the competition of English ball bearings which were than recognized as of standard precision.

However, none of these bearings were satisfactory. The German government financed the research which led to improvements. This research was entrusted to Dr. Stribeck whose work was thorough and comprehensive. He determined that the balls must be microscopically exact in size or some balls would do all the work while others were useless. He also laid emphasis upon the necessity of excluding all dirt and grit that might roughen the rolling contact surfaces and thereby diminish their effect. The result of the Stribeck investigation set the ball bearing industry on a scientific basis.

When the famous Fafnir Company was formed in 1909, two years were devoted to an intensive investigation of the problems of manufacture before a single ball was offered on the market. Since that time the company has been rewarded by gaining a place among the leaders of high grade non-friction bearings.

The demonstration in our Museum furnished by the SKF Industries, Inc., shows conclusively the precision in size, the equality of hardness, and the regularity of finish in commercial bearings. Two hundred balls bounce in continuous sequence over a series of anvils into a central container. From this they are sent on a further journey over the anvils, to the astonishment of the beholder.

NOTES FROM THE FRANKLIN INSTITUTE LABORATORIES FOR RESEARCH AND DEVELOPMENT

NICOL H. SMITH, DIRECTOR

SUBSTRUCTURE IN METALS

BY

R. L. SMITH

Substructure in metals is the structure within grains or single crystals. Generally a poly-crystalline piece of pure metal is thought to be made up of an aggregate of grains and that these grains are nearly perfect crystalline arrays of atoms. In the case of single crystals where there are no interfering grain boundaries, the regularity of the arrays is considered to be uninterrupted throughout the specimen. This regularity, of course, has slight imperfections introduced in it by virtue of the presence of impurity atoms, vacancies, or dislocations.

In many cases, special etching techniques reveal the presence of a structure within a single grain. This structure may manifest itself by irregular wavy lines such as the "veins" in ferritic steel or it may occur in a regular closed pattern such as shown in Al and Zn.

If a relatively pure iron is heat treated above 723°C. it undergoes a volume change. Hultgren and Herrlander (1) postulated that this volume change causes plastic deformation within the grain that in turn produces a sub-grain structure commonly known as "veining."

The importance of such a substructure has often been overlooked. Considerable effort has been directed toward evaluating the effect of grain size on the physical properties of metals with complete disregard for the effect of a smaller structure within the grain. This substructure is important in affecting mechanical properties, especially at low temperatures. For instance, the author has shown that the yield stress in low carbon Fe-C alloys is affected considerably by the presence of substructure as manifested by "veining" (2). At normal temperatures a low carbon (about 0.04 per cent or less) alloy has a yield stress that is dependent primarily on grain size. The smaller the grain size the higher the yield stress. At liquid air temperature this relation no longer holds although yield stress will increase with decreasing grain size. A large yield stress variation can occur at constant grain sizes by the introduction of sub-grain boundaries. This variation may be due to the interference of dislocation movements by the sub-grain boundaries.

Work in progress in this laboratory by Jaumot and Smith (3) shows that diffusion results are affected considerably by the presence of substructure. In single crystals of zinc a substructure appears which is not revealed by back reflection X-ray techniques, since the amount of disregistry between the sub-grains is only a matter of a few seconds of arc. Special etching techniques reveal such a structure. This structure has resulted in anomalous self-diffusion values. At low temperatures grain boundary diffusion becomes more important than volume diffusion. Work in The Franklin Institute Research Laboratories is being conducted on the evaluation of the effect of a substructure on self-diffusion in zinc since it seems reasonable to assume that sub-grain boundaries should also act as easy paths for diffusion.

It has been postulated that sub-grain boundaries are made up of arrays of edge dislocations. Vogel *et al* (4) showed that the lineage boundaries in germanium could be etched in such a way as to show discrete etch pits at relatively constant distances from each other. When this distance was measured and the orientation of the subcrystals on either side of the lineage boundary taken, it was found that the dislocation model of the sub-grain boundary was verified experimentally. Steijn (5), in this laboratory, has recently verified the dislocation model for zinc which, as mentioned above, exhibits a cellular substructure for single crystals grown from the melt.

Considerable work is in progress to determine the detailed nature of the sub-grain boundary and to evaluate its effect on the properties of the metal in which it is formed. The presence of substructures should always be looked for in metallurgical experiments and its possible effect should be considered.

REFERENCES

(1) HULTGREN AND HERRLANDER, *Trans. A.I.M.E.*, Vol. 172, p. 493 (1947).

(2) R. L. SMITH, G. SPANGLER AND R. M. BRICK, *Trans. A.S.M.*, Vol. 46, p. 973 (1954).

(3) To be published.

(4) F. L. VOGEL, W. G. PFANN, H. E. COREY AND E. E. THOMAS, *Phys. Rev.*, Vol. 90, p. 409 (1953).

(5) To be published.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

ELLICE McDONALD, DIRECTOR

SYNTHESIS OF SOME 2-OXO-4-ARYL-5-CARBETHOXY-6-TRIFLUOROMETHYL-1,2,3,4-TETRAHYDROPYRIMIDINES

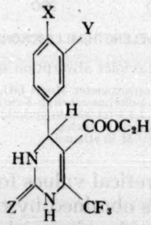
BY

HENRY A. RUTTER, JR. AND LILLIE O. GUSTAFSON

In a search for new compounds of value in tumor chemotherapy, it was decided to prepare a series of pyrimidines containing fluorine. The small size of the fluorine atom should not prevent access of the pyrimidine molecule to an enzyme surface and the high electronegative nature of fluorine should markedly influence metabolic attack of the ring by biochemical oxidation-reductions. Nadel and Greenberg (1) observed that a 2,4-diamino-5-aryl-6-methyl-pyrimidine was inhibitory alone and in combination against leukemia in mice. Analogous compounds containing a trifluoromethyl group in place of the 6-methyl group could possibly retain antitumor action and not be easily reversed by metabolites just as perfluoro compounds are inert in many chemical reactions.

The series of compounds selected for this investigation, 2-oxo-4-aryl-5-carbethoxy-6-trifluoromethyl-1,2,3,4-tetrahydropyrimidines were prepared essentially by the method of Folkers *et al* (2). Urea, 0.1 mole, 0.15 mole ethyl trifluoroacetoacetate, and 0.1 mole of the appropriate

TABLE I.



X	*Y*	*Z*	Yield, %	M.p., C° (uncor.)	Nitrogen, % Calcd.	Nitrogen, % Found	Sulfur, % Calcd.	Sulfur, % Found
H	H	O	25	165	8.92	8.63	—	—
CH_3O	H	O	30	156–7	8.13	7.99	—	—
C_2H_5O	C_2H_5O	O	25	149–50	6.96	6.47	—	—
$N(CH_3)_2$	H	O	20	169–70	11.76	11.07	—	—
H	H	S	30	180–180.5	8.51	8.19	9.70	9.65

substituted benzaldehyde in 25 ml. of glacial acetic acid plus 4 drops concentrated hydrochloric acid, were heated in a water bath for 3 hours. The solution was cooled and poured into cold water and the thick oil washed by decantation, as much water as possible being poured off. The residue was stirred and allowed to stand under isopropyl ether until a solid was obtained, about one hour. The solid was collected on a Buchner funnel and washed with isopropyl ether. Finally the solid was recrystallized from 50 per cent ethanol with added Norit-A until the crystals reached constant melting point.

The 4 new compounds were soluble in ethanol, insoluble in isopropyl ether, and difficultly soluble in hot water. The results of the Kjeldahl analysis agreed with the theoretical values for nitrogen. No quantitative estimation of fluorine was obtained by the use of sodium in liquid ammonia, Parr bomb fusion, or heating with calcium oxide.

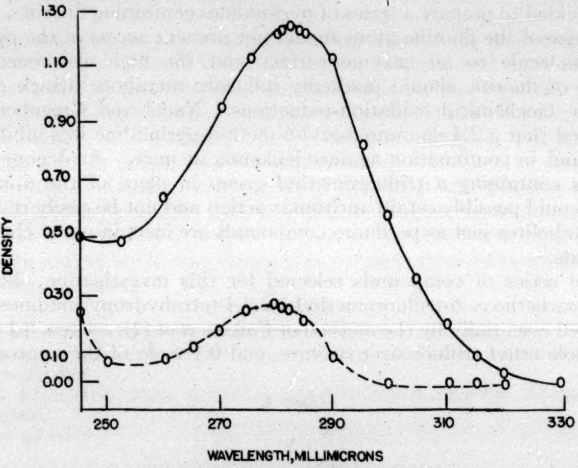


FIG. 1. Ultraviolet absorption spectra.

Beckman Spectrophotometer Model DU, 1 cm. cell.
—— 2-oxo-4-(3,4-diethoxyphenyl)-5-carbethoxy-6-methyl-1,2,3,4-tetrahydropyrimidine.
---- 6-trifluoromethyl analog.
Solutions 1×10^{-4} M in ethanol.

Final proof of structure of the assumed pyrimidine ring in these substances was obtained by three investigations.

First, a thio analog was prepared by a condensation using thiourea in place of urea. This compound as shown in Table I, where *X*, *Y*, and *Z* are the elements or groups indicated, has the correct analysis for nitrogen and sulfur expected on the basis of the assumed structure.

Sulfur analysis was obtained by the perchloric-nitric acid method of Wolesensky (3).

Second, the presence of the 5,6-double bond was proved by the reaction of 2-oxo-4-(3,4-diethoxyphenyl)-5-carbethoxy-6-trifluoromethyl-1,2,3,4-tetrahydropyrimidine with 30 per cent hydrogen peroxide in concentrated hydrochloric acid at room temperature yielding an addition compound, Mp 141–142.5, with the hypochlorus acid (Cl 7.9%, calcd. 7.87; N 6.08%, calcd. 6.21).

Finally, the original pyrimidine was compared for ultraviolet absorption spectrum with a known pyrimidine which was identical in every respect with the exception that a 6-methyl group was present instead of the 6-trifluoromethyl group of the former.

As shown in the absorption curves, Fig. 1, the absorption peaks occur at 280 mμ and 282 mμ respectively. The difference in density is probably due to the difference in the substituents at the 6-position of the pyrimidine rings as shown, for example, at 300 mμ where the densities are 0.006 and 0.596, and at 290 mμ are 0.107 and 1.14 respectively.

The data taken collectively indicate the synthesis of a pyrimidine ring system containing the trifluoromethyl group.

REFERENCES

(1) E. M. NADEL AND J. GREENBERG, *Cancer Research*, **13**, 865 (1953).

(2) KARL FOLKERS, H. J. HARWOOD, AND TREAT B. JOHNSON, *J. Am. Chem. Soc.*, **54**, 3751 (1932).

(3) EDWARD WOLESENSKY, Vol. 1, "Perchloric Acid," Third edition, Columbus, Ohio, G. Frederick Smith Chemical Company, 1934, p. 41.

BOOK REVIEWS

Rare Metals Handbook, edited by C. A. Hampel. 657 pages, illustrations, 16 × 24 cm. New York, Reinhold Publishing Corp., 1954. Price, $12.00.

This text is a metals handbook covering 34 uncommon metals. Many of the metals discussed are not particularly rare but are used in relatively small quantities as compared to such metals as iron, aluminum or copper. Some are elements that are relatively abundant in the mineral deposits of the earth, but are at the present time so expensive to separate from their ores that they have not been used extensively. An important example is titanium.

A handbook of this type is subject to criticism as to the choice of material and the space allocated to any given metal; however, the editor has done an excellent job in choosing the metals to be covered and in standardizing the individual author's treatment. The general treatment for each metal is to present a brief historical sketch, followed by the production from the ores, the chemical and physical properties, fabrication techniques, and uses. Extensive bibliographies are presented at the end of each chapter. For instance, the chapter on cobalt has 300 references and that on uranium has 389. The chapters range in length from ten pages for silicon to 72 pages for uranium.

In addition to the sections on the individual metals there is a final twenty page summary of the physical properties of metals. This summary presents considerable data on the physical properties of both the common and uncommon metals and thus allows one to compare conveniently the properties of several metals. The author realizes the difficulty of obtaining meaningful data on some of the metals that are difficult to obtain in pure form, and takes great care in pointing out that in some cases the values given must be considered only as representative of the properties of the particular metal.

One disappointing feature of this book is the lack of constitutional phase diagrams. Only two chapters contained a list of diagrams. These were the sections on lithium and titanium. The extra space required would have been well justified by such data. The chapter on titanium presented fifteen phase diagrams on two pages. These diagrams enhanced the value of this chapter considerably.

A 17-page section on the fifteen rare earth metals is rather brief in its treatment of the individual metal. It is difficult to obtain information on these elements and a detailed treatment would have been very useful.

Notwithstanding the preceding minor criticisms, this book is excellent. It should be acquired by anyone who finds the well-known A.S.M. Metals Handbook useful. It will undoubtedly take its place as one of the most important reference books in the metals literature.

R. L. Smith

Selected Combustion Problems. Fundamentals and Aeronautical Applications, by the Advisory Group for Aeronautical Research and Development of the North Atlantic Treaty Organization. 534 pages, diagrams, 16 × 25 cm. London, Butterworths Scientific Publications, 1954. No price given.

Since World War II aeronautics has become thoroughly revolutionized. Jets and rockets have made reciprocating engine aircraft obsolete for military purposes and today are threatening to displace the conventional airliner. In the not too distant future the gas turbine may even take the place of the spark ignition engine in automobiles.

These developments are reflected in the increasingly larger number of publications on combustion and related phenomena per year. It is therefore of great help to people working in this field to have the papers presented during symposia collected in one volume rather than distributed among numerous scientific journals. The papers of the Third and Fourth International Symposia on Combustion, held in this country in 1950 and

1952 respectively, have therefore been published in book-form; the volume reviewed here contains the papers presented at the Combustion Colloquium at Cambridge University in December 1953 and is, in fact, a continuation of the other two volumes. The Advisory Group for Aeronautical Research and Development of NATO (known as AGARD), under the direction of Theodore von Kármán, is to be congratulated for the promptness and care with which this book was published.

The purpose of the Colloquium (and of the preceding Symposia) was to promote an exchange of ideas between the researchers on combustion and the engineers responsible for engine development. Thus, discussions of research on laminar flame propagation, turbulent combustion, and liquid and solid propellants are followed by a discussion of technical combustion problems. The closeness of theory and practice, one of the characteristics of this field of combustion, is thus shown.

Laminar flame propagation (Section I) is probably the field of combustion study most amenable to theoretical interpretation. Yet even there we are still far from a satisfactory and readily applicable theory. Physically, both diffusion of reacting material and heat conduction have been shown to play important parts in flame propagation; chemically, chain reactions must be involved but there is no agreement at all as to their exact nature and relative importance. Accordingly, Kármán and Penner's theory takes into account both diffusion and heat conduction and reduces all the chemical reactions to only one "global," i.e. rate controlling, step. In the case of hydrazine, their assumptions appear to fit the actual situation and they are able to calculate burning velocities in excellent agreement with those observed. While this theory is a step forward, it is too limited in applicability to be of much practical value. Other authors attach different weight to diffusion and heat conduction than do Kármán and Penner, but these differences do not seem too important. They all postulate a global reaction, at least in their examples. None of them tries to determine the chemical kinetics from the observed flame velocity.

While the burning velocity is the most characteristic property of laminar flames, temperature and concentration profiles are probably just as important for their understanding. They are, however, much more difficult to obtain. This aspect of the problem is very well reviewed by Linnett. Gaydon discusses the significance of the results of his spectroscopic investigations of flames on the elucidation of chemical reaction mechanisms. Peroxides and aldehydes, generally assumed to be important links, are not found spectroscopically even in premixed flames. On the other hand, C_2 and CH, which are not usually accorded a place in postulated mechanisms, are shown to be present in abundance. Gaydon concludes with the opinion which has become the present trend of thought, namely, that in premixed as well as in diffusion flames the first step is the breakdown of the saturated hydrocarbons to unsaturated molecules or radicals, this step being followed by the liberation of carbon and hydrogen and the formation of oxidized products. Penner gives a review of spectroscopic studies of premixed laminar flames. He emphasizes the application of spectroscopy to the determination of flame temperatures and deals at some length with the arguments for and against anomalous rotational temperatures, a phenomenon which has been widely investigated.

In the minutes of the discussion following the presentation of these papers, Wohl reports results of his spectroscopic temperature determinations of hydrogen flames, in which he used the intensity ratio of two peaks of the water vapor bands in the near infrared. The burning zone is characterized by $T_{observed} > T_{adiabatic}$; i.e. by an excess enthalpy as postulated by the well-known Lewis-von Elbe theory.

Turbulent combustion (Section II) is, of course, very important to us because it constitutes a means of increasing effective flame velocity. Bernard reviews some of the results and the classical methods of studying this subject; Scurluck and Grover, and Karlovitz present their theories of turbulent combustion. In spite of great differences of detail, both theories use the concept of the wrinkled surface and the theory of turbulent diffusion for computing the root-mean-square displacement of the flame front and introduce flame generated turbulence as a sort of perturbation. Among the experimental work described Karlovitz' measurement of flame brush thickness by an electronic probe shows particular ingenuity. Essentially it consists of counting the number of times the flame front passes a given point by making use of the high electric conductivity of the combustion zone.

In the section on solid and liquid propellants (Section III) Adams and Wiseman discuss combustion of double-base propellants, Geckler the mechanism of the combustion of solid propellants, Spalding the combustion of single droplets and of a fuel spray, and Ross and Datner the combustion instability in liquid propellant rocket motors. These papers are quite inclusive and direct attention to the most important open questions. It is significant that we are still in the dark e.g. as to whether droplet burning or jet mixing processes are predominating in aircraft engines where it is common practice to inject liquid fuel directly into the combustion chamber, or how a solid propellant passes into the zone of active combustion (by a solid state reaction or by fusion and subsequent reaction). An answer to these questions seems important, for if e.g. a solid propellant has to fuse first and a solution is formed, control of the melting point by choice of the lowest melting component and of solubilities could modify the properties of the propellant in the desirable direction. More serious than these problems is that of combustion instability which has been the cause of many aircraft failures. Our lack of understanding makes it necessary to test a prototype of each engine, not a small-scale model but a full-size one, for we do not even know how to scale. Ross and Datner point out that combustion oscillations may be excited by disturbances of the random noise type, inherent in the combustion, or by a shock such as a "hardstart." They put special emphasis in the combustion time lag (Rocket engines are dynamic systems with delayed reaction).

In the final section which deals with technical combustion problems (Section IV), combustion at high altitudes is discussed by Lloyd and Mullins, the stabilization of flames by obstacles by Barrère and Mestre, combustion in vitiated air by Mullins, and combustion in pulse-jets by Bertin. All these papers are very well and interestingly written; that by Mullins should also be immediately useful to the designing engineer for its numerous nomograms relating essential combustion parameters. Bertin also studied the effect of additives, but found no appreciable improvement in over-all performance of pulse-jets. Barrère points out a striking parallelism between the performance of a compound in a pulse-jet and its ignition delay as determined by Mullins for conditions simulating those prevailing in a gas turbine.

Clearly, the book contains a wealth of information. While the papers have not been edited to the extent of conforming to a uniform notation throughout, each theoretical paper has an explanation of the symbols used at the end. Everyone active in combustion work will find this volume invaluable. James L. Lauer

Introduction to Astronomy, by Cecelia Payne-Gaposchkin. 508 pages, diagrams, illustrations, 16 × 24 cm. New York, Prentice-Hall, Inc., 1954. Price, $6.00.

In her preface the author states: "My chief reason for writing the book was a desire to give emphasis to stars and stellar systems as well as to the solar system, which usually occupies the major part of an elementary book. The greater part of modern astronomy is concerned with stars, and even an introductory text should reflect this emphasis."

The reasons are sufficient and the end product fulfills her goal. In *Introduction to Astronomy* Dr. Cecelia Payne-Gaposchkin, Phillips Astronomer at Harvard University, has written a thoroughly readable and articulate book which this reviewer will use.

Textbooks on astronomy are common; good ones are rare. Here is an example of a neatly balanced book which can be used in a one or two semester course. This reviewer likes the balanced prospective of the author and finds her choice of material excellent. The distinguishing mark about a book is not what is in it but what has been omitted. To this end Dr. Gaposchkin has demonstrated an uncanny skill in weeding out the extraneous material without cutting the important details. As usual, her facile pen has painted a stimulating picture with an enviable ease.

Roughly half the book is devoted to the solar system with separate chapters dealing with the earth, sun, moon, planets and asteroids. However, in the same part of the book and separated from the members of the solar system are found chapters on "Tools and Methods" which deal with celestial distance finding, the physics of atoms and the instruments of modern day astronomy. In the chapter on "The Solar System" are found the laws of celestial mechanics under which the system operates and the evolution of these fundamental laws from ancient days.

The second half of the book deals with the stellar systems in which the author is a dominant force on the contemporary scene. Here are chapters on the geometrical properties of the stars, the physics of the stars, the variable stars which is such an important facet of present stellar research and finally the systems of the stars—first in the galaxy and then in systems of galaxies. This is one of the most fascinating and comprehensive treatments this reviewer has ever encountered.

Another excellent point in this book is the treatment of the evolution of the solar system and the origin of things. These topics have been investigated by some of the most skillful and resourceful scientists of our time. The author has, in two chapters, brought the reader up-to-date in the current thinking on these topics. These chapters are skillfully written and will appeal to the student.

Introduction to Astronomy is the best modern textbook on astronomy in print today.

I. M. LEVITT

WIND TUNNEL TESTING, by Alan Pope. Second edition, 511 pages, illustrations, diagrams, 15 × 22 cm. New York, John Wiley & Sons, Inc., 1954. Price, $8.50.

The first edition of this book, issued in 1947, presented a concise but complete analysis and study of the scattered but vast material on the subject; and a lucid presentation was given on the design of a tunnel and propeller-flow straightener system. The procedure for testing a new model airplane was described in detail, including corrections, extrapolation to full scale, etc.

In this second edition, the author states, "The purpose of the second edition of *Wind Tunnel Testing* is twofold: to revise the data and procedures given in the first edition when changes are justified in the light of new knowledge, and to extend the scope of the book to embrace new techniques not available when the first edition was prepared." The author is to be commended on his well balanced and up-to-date presentation on the wealth of material in this rapidly expanding and changing field. He has correlated and organized the scattered technical information into a unique and welcome new addition to the aeronautical library.

New chapters include the subjects of testing at nearsonic, transonic, supersonic and hypersonic speeds, the testing of helicopter rotors, and a discussion of experimental ways for using wind tunnels in non-aeronautical capacities. The total of twelve chapters includes: (1) The Wind Tunnel, (2) Wind Tunnel Design, (3) Instrumentation and Calibration of the Test Section, (4) Model Force, Moment and Pressure Measurements, (5) Testing Procedure, (6) Wind Tunnel Boundary Corrections, (7) The Use of Wind Tunnel Data, (8) Small Wind Tunnels, (9) Non-Aeronautical Use of the Wind Tunnel, (10) Rotor Testing, (11) Nearsonic and Transonic Testing, and (12) Supersonic Wind Tunnel Testing.

Noteworthy also are the excellent problems and illustrative examples showing the theoretical considerations, applications and limitations of theory with experimental and actual flight test results.

SAMUEL M. BERKOWITZ

RESIDUAL STRESSES IN METALS AND METAL CONSTRUCTION, edited by W. R. Osgood. 363 pages, diagrams, 16 × 24 cm. New York, Reinhold Publishing Corp., 1954. Price, $10.00.

Welding is a very rapid method of fabricating steel. However, it induces residual stresses due to heating and subsequent cooling and unless proper precautions are exercised, cracks may develop in the structure. During the last war, in order to produce as rapidly as possible, many ships were constructed by various means of welding. When some of these so-called Liberty ships cracked in half, or failed in less spectacular fashion, an extensive research program was initiated by the Department of the Navy to determine the causes of the failure. As a result of this research, a great deal of information was accumulated. In order that these data and other information as well, be assembled, the Ship Structures Committee (composed of several member agencies of the Government) directed the Committee on Residual Stresses of the National Academy of Sciences and the National Research Council to prepare a monograph on the significance of residual stresses on the strength of structures, with particular emphasis on their effect on fracture. *Residual Stresses in Metals and Metal Construction,* edited by W. R. Osgood of the University of Illinois, is the result.

The book is composed of 22 papers prepared by recognized authorities in the field of residual stresses. A summary prepared by members of the Ship Structures Committee highlights the significance of the various papers and concludes the book. It also suggests additional research to learn more of the complicated nature of effects of residual stresses.

In subject matter, the papers range from theory to practice, from methods of measuring residual stresses to the effects of residual stresses on structures. A number of the papers such as "Effects of Residual Stresses on the Behaviors of Structures," "Residual Stresses in Welded Pressure-Vessel Manufacture" and "Residual Stresses as a Factor in Ship Fractures" deal with specific structures. Other papers such as "Residual Stresses in Welding," "The Significance of Residual Stresses" and "Some Remarks on the Influence of Residual Stresses on the Brittle, Plastic and Fatigue Behavior of Structures" deal with the general effect of residual stresses. And still others, such as "Measurement of Residual Stresses," "Complete Determination of the State of Residual Stress in Solid and Hollow Metal Cylinders" and "Determination of Residual Stresses Below the Surface," deal with the measurement of residual stresses. Unquestionably the subject of residual stresses is thoroughly explored. It is interesting to know, too, that there is still disagreement among the experts as to the real effect of residual stresses upon the strength and performance of a structure. Some believe they affect the structures very little; others believe their effect is quite important.

For anyone interested in this matter of residual stresses and this should apply to those engineers concerned with the strength of structures, this book should be very valuable. And not only for the papers in themselves, but for the many references to the subject contained at the end of each chapter.

E. W. HAMMER

THE MOON PUZZLE, by N. O. Bergquist. 378 pages, diagrams, 13 × 19 cm. Copenhagen, Grafisk Forlag, 1954. Price not given.

The acute problem of the "birth of the Moon" has occupied the scientist for a long time. About the middle of the last century the English scientist G. H. Darwin postulated that the moon was born from the earth while the earth was still in a plastic state and the spot from which the moon was torn became the great Pacific basin. As time went on, and new mathematical tools became available, to subject this theory to analysis, it was seen that the mechanism was an impossible one. Later other theories were propounded and these too were incapable of substantiation. Today, the question is still unanswered.

In *The Moon Puzzle* N. O. Bergquist, a Swedish engineer, attempts to account for the birth of the moon and many of the surface features of the earth by postulating that in comparatively recent geologic times an intruder object, the author calls it a planetoid, plowed through the earth beginning at the western edge of what is now the Pacific Ocean and moved east and south over the surface of the earth. The mechanism of the encounter and the results provide material for a book of over 300 pages.

In his book the author tries to account for other theories which have recently been presented to us and which are also most difficult to subject to analysis and prove or disprove. He, for instance, takes up the spreading of the Atlantic Ocean with the shapes of the eastern and western hemispheres fitting like pieces of a puzzle. This is the old Wegener hypothesis. He accounts for the magnetic field of the earth, the

Deccan massif of India, the basaltic plateaus around the world, the changes in the flora and fauna and even accounts for the extinction of the dinosaurs to mention a few mysteries of science which he believes is explainable by his hypothesis.

The book is rather difficult to read for it deals with so many facets of the problems and no one specialist can possibly judge the accuracy of the entire book. It would be a long tedious job for a group of specialists.

In the astronomical field, this reviewer finds some objections to the astronomical implications.

For instance, on page 18, the author indicates the absence of tides because the moon was not yet born. Actually, the tides of the sun are about half the magnitude of the lunar tides so these cannot be neglected.

On page 19 while the tilt of the axis of the earth produces seasons the eccentricity of the earth's orbit also has an effect. In the absence of the first the second would be dominant, albeit small.

On page 80 no mention is made of the reaction effects when ". . . the active forces shot the 'skull-cap' away in a great volcanic eruption, imparting to the newborn Moon a certain vertical speed relative to the earth." This action must have had a reaction—what was it?

On page 166 present day astronomers still refuse to believe that the axis of the earth was tilted by a collision or that its rotational speed could be altered. The effects are too catastrophic if a body of the size and speed necessary to effect this change actually came close enough to attempt it.

Throughout the book the author lists astronomical objections to his theory which he claims will partially reject or modify the hypothesis but he sees no reason for its complete rejection.

Only the reader can determine whether the arguments presented in *The Moon Puzzle* are valid and whether they answer the claims made for it by the author. I. M. LEVITT

CURRENT TOPICS

Ultra-High-Frequency Transistor.—The time when home television sets will use tiny transistors about the size of pencil erasers instead of a large array of vacuum tubes came closer to realization with the announcement by Bell Telephone Laboratories of a transistor which will operate at 440 million cycles, higher in frequency than any transistor yet known.

Bell Laboratories, where the original transistor was invented, unveiled its latest member of the family of the "mighty mites" with a talk by Dr. J. M. Early, before the Western Electronics Show and Convention at Los Angeles, Calif. Dr. Early invented the new transistor and was assisted in its development by W. C. Hittinger and Dr. J. W. Peterson.

Bell scientists foresee its use in portable radio sets, the transcontinental radio relay system and submarine telephone and television cable repeaters. Used in military equipment, the device could be of tremendous importance to national defense.

The ultra-high-frequency device, called an "intrinsic barrier" transistor, can increase an electrical signal a thousand-fold. In addition to its ability to amplify signals, it may be used as a generator of electrical oscillations. Frequencies as high as 3,000 million cycles per second are theoretically possible.

The new transistor builds on the basic theory that resulted in Bell Laboratories' first transistor and the recently announced Bell Solar Battery. Constructed entirely of solid materials, transistors, unlike vacuum tubes, have no vacuum to maintain. They are "cold"; there is no need for a heater to boil off electrons, therefore they respond immediately to an electrical impulse. They are rugged, and have a long life expectancy since the parts do not wear out in use.

A junction transistor is an electrical "sandwich" made by creating, in a germanium crystal, a very thin region of one electrical type which separates two adjoining regions of a different electrical type. Bell scientists have now made, in effect, a "club sandwich" by adding an intrinsically pure layer to this basic, positive-negative-positive wafer which acts as a "shock absorber" for high voltage electrical charges. This chemically pure part of the germanium crystal remains neutral, while three adjoining sections become positively or negatively charged. The "intrinsic" region in the midst of the transistor permits the faster movement of positive charges, isolates the input and output areas, and reduces the stored energy to make functioning at higher frequencies possible. The increased separation of input and output areas also permits operation at higher voltages than was possible in earlier transistors.

Two tiny dots of indium, a low-resistance metallic element found as an impurity in zinc blends, are connected to the opposite faces of the n-type-intrinsic wafer to provide input and output electrodes.

Transistor action, in general, depends upon the fact that electrons in semiconductor materials, such as germanium, can carry current in two distinctly different ways. This is because most of the electrons in a semiconductor do not contribute to carrying current at all. Instead, they are held in fixed positions and act as a rigid cement to bind together the atoms in a solid. Only if one of these electrons gets out of place, or if another electron is introduced in one of a number of ways, can current be carried. If, on the other hand, one of the electrons normally present in the cement is removed, then the "hole" left behind it can move like a bubble in a liquid and thus carry current in this "musical chairs" arrangement.

When electrical current is applied to an intrinsic barrier transistor, "holes" introduced at the positive section of the transistor at low voltage spread across the negative region at increased speed. The "holes" then drift at maximum high velocities through the neutral region to the output section of the crystal. There, at greatly increased voltage, they are collected to provide useful power output and gain.

The new device, like other junction transistors, can operate at extremely low power—as little as a fiftieth of that used by an ordinary flashlight bulb and less than a hundredth of that needed by a vacuum tube. But, unlike earlier transistors, it can also operate at relatively high power, perhaps up to one-half a watt. Like other transistors, it is expected to have a very long life since its parts do not wear out in use. It can also withstand as much as 100 volts.

Although the new transistor is still in the experimental stage, frequencies as high as 440 million cycles per second have been generated to date. This is well into the ultra-high-frequency range used by FM radio and TV.

Insect Control Equipment.—Most of us are concerned with insect control only a few months in the year. With the Army, though, it's a year long and a world wide proposition. Insect-borne diseases may put a fighting man out of action as effectively as a human enemy. Nor is this threat confined to the tropics. Short Arctic summers bring with them short-lived but voracious mosquitoes.

At the Engineer Research and Development Laboratories, Fort Belvoir, Va., the problem is being attacked from the equipment angle. Here the design and development of equipment used in combat zones and communications areas for the ground dispersal of insecticides are accomplished. This poses problems quite different from those encountered in the development of commercial equipment. In addition to effectively destroying insects, military equipment must be rugged, simple to operate, easy to maintain and able to withstand temperature extremes.

A two-gallon pack-type sprayer, one of the Laboratories' most recent developments, was designed to meet these requirements. Fabricated of stainless steel, found to be more corrosion resistant than the coated type, it incorporates many hardened aluminum parts to reduce weight. Sturdy construction gives it a life span up to five years. It is used principally in production of a wet spray for applying residuals of insecticides to walls and screens for killing adult insects, and for the application of fine mists to larval breeding areas.

Among the items currently under test at ERDL are a fog generator, a mist blower and a hydraulic sprayer. The fog generator, an air compressor type with a five-gallon capacity, can also produce insecticide mist, depending upon the nozzles used. Insecticide fog is employed chiefly indoors. The mist is effective in the control of mosquitoes and flies.

Suspensions, solutions and dry insecticides may be used in the 80-gallon capacity mist blower. This method is particularly effective in combating insect life in thick foliage.

The hydraulic sprayer under test weighs one-quarter as much as the present Engineer model. It is used for spraying tall trees, soaking vegetation and under buildings for termite control.

Development and test are planned to insure the maximum in proper design and in adaptation to the needs of the ultimate user. Pilot models are built after careful analysis of the findings of others engaged in similar projects and of the reports of armed forces units in the field. Engineering tests are conducted at Fort Belvoir and at field stations at Fort Churchill, Canada and Orlando, Florida. These ascertain the volume and rate of output, particle size range, ease of operation, and suitability for military use. Tests are also made under extremes of heat and cold, shock, and exposure to corrosive materials.

Following these tests, the equipment is subjected to all the usual hazards of military use by Preventive Medicine units of all the armed services. Based on the results, drawings and military specifications are prepared by the ERDL Insect and Rodent Control Section.

Macro and Micro Combination Viewer-Projector-Camera.—"Projectina," a Swiss precision instrument, provides individual or comparative study, measuring, counting, testing, recording of transparencies and opaque objects through macro (close-up) and microscopic eyepiece viewing, projection (on built-in or wall screen) and photography. It is an excellent visual aid for instruction and documentation as well as research and quality control in science and practically any industry (metal, machinery, watchmaking, electrical, paper, leather, food, chemical, etc.).

Eyepiece viewing includes 3 interchangeable oculars and 3 micro objectives (which can be supplemented by standard oculars and objectives of other makes) plus a special cross-hair eyepiece. Projection features include groundglass observation (with 2 supplementary projection objectives), 2 cross-hair graph screens for precise measurement readings, circular scale for determining angles, as well as wall-viewing for group study. Photographic reproduction is achieved through the use of available 4 × 5-in. filmholder carrier interchanged with groundglass, or a single-lens reflex camera which is attachable to eyepiece mount.

The 3 micro objectives (2 of which are interchangeable) are mounted on a revolving dial, and in combination with varied eyepieces or projection objectives provide magnification from 10 up to 1500 times. Transmitted-light system for the study of transparent material (micro slides) features built-in, adjustable 30 Watt lamp with revolving double condensers, diaphragm and a range of 3 intensities. Reflected- or surface-light system for opaque objects includes condenser and 2 intensity ranges, features similar lamp on 3-joint side-arm, adjustable to any position. Both systems may be combined for simultaneous examination of silhouette and surface, creating 3-dimensional relief effect.

Three-way focusing adjustment is made possible by 3 × 4-in. removable object table, controlled by micrometer screws with zero marks. Scales on table easily determine precise positioning. Revolving dial below object table has provision for 4 filters: Green filter relieves observer's eyestrain; orange filter increases contrast for screen readings; polarizing filter with compensator permits differentiated microscopic inspection; a fourth aperture accepts infra-red or any photographic filter.

Stroboscopic Discs.—It has been suggested at the Boston Naval Shipyard that stroboscopic discs be manufactured in the shop and used to set the speed governors of motion picture projectors. If made correctly (each for a specific rpm.), the discs can be used for accurate settings. A 60-cycle frequency fluorescent or ¼ watt neon lamp serves as a light source. The discs can be used as a substitute stroboscopic timing device at facilities where the faster and more nearly accurate stroboscopic tachometer is not available.

Air-Sea Rescue Drift Buoy.—Under contract with the Office of Naval Research, Woods Hole Oceanographic Institution has developed a device, the Air-Sea Rescue Drift Buoy, to aid in locating survivors of naval aircraft and vessels abandoned at sea. With modern devices, survivors of disasters at sea can transmit the location of the accident, but winds and currents bear the emergency craft away from the accident site before the rescue craft appear. Detailed on the spot broadcasts of weather conditions, winds and currents would enable modern electronic wizards to compute drift and give the location of the survival craft any given time. Office of Naval Research Scientists, however, believed development of a satisfactory air-sea-rescue device would be simpler than solution of the complex weather problems.

New Camera Holder.—The "Sholderpod" has just been introduced by the Monu Inventions Corporation of Englewood, Colorado. The new camera holder, designed to replace the tri-pod, is a precision built, adjustable camera platform made of light weight aluminum. This new camera accesory derives its name from the fact that the shoulder of the photographer is used to steady the camera. A curved aluminum piece, shaped like a half "U," fits snugly over the shoulder. The adjustable extension has a pistol grip fitting at the other end which gives maximum flexibility and steady shooting. Quickly taken apart, the "Sholderpod" can be stowed in most any camera case or gadget bag.

While new on the market, those who have used it have been quick to point out the many advantages of the "Sholderpod." It has improved the quality of many home movies, especially where "panning" is involved, by eliminating the excessive motion generally found in such efforts. Very handy for use with heavier type press cameras as the weight of the camera is on the shoulder. Used by many professionals when shooting at slower shutter speeds.

New Method of Bonding Fibreglass.—The Bureau of Ordnance has announced the development of a new structural material for bonding fibreglass, which will produce laminates and tubular products of superior strength. Instead of the conventional method of bonding fibreglass with resins and organic materials, the new methods combine metals and inorganic substances, or combinations of both. Fibreglass filaments are formed in the usual way by the use of appropriate refractory furnaces and bushings. As the filaments emerge from the furnace at high speeds, they are individually coated with metallic and other inorganic substances. Metal coatings, such as iron, nickel, molybdenum, aluminum, zinc, lead, tin, and copper, have been applied. The coated fibres are then bonded and compacted to form tubular products and laminates in a subsequent stage of the process which involves the application of suitable conditions of heat and pressure with or without the use of the more common flux materials. The new process, on the basis of work accomplished to date, indicates that the following important advantages may be obtained: (1) A fibreglass reinforced structural material which is suitable for higher temperatures than presently available fibreglass structural materials, (2) Improved strength-weight ratio, (3) High resistance to usual corrosion agents, (4) Raw materials available in this country. When available, the new material should have important commercial application. The Bureau of Ordnance is seeking industrial participation in the further development of the new material and in the determination of its uses.

Post-Irradiation Treatment of Animals.—Pulverized particles of limestone, quartz or glass injected into animals after moderately high doses of radiation to the whole body increase their chance of survival. This finding was reported to the American Physiological Society Fall Meeting by Dr. Falconer Smith, Dr. Willie W. Smith, and Dr. Howard L. Andrews of the National Cancer Institute, Public Health Service, U. S. Department of Health, Education, and Welfare.

There are only a few post-irradiation treatments known which are of benefit to animals with radiation sickness, Dr. Falconer Smith said. He explained that the injection of these foreign particles into the animals causes a temporary inflammation which in some unknown way brings about the survival of one-fifth of the animals which would otherwise die after exposure to radiation.

The scientists found that as the size of the particles was reduced the beneficial effect was improved. They got as good protection in the animals given particles under the skin as in those injected with particles in the peritoneal cavity, Dr. Smith reported. Mice, guinea pigs, and hamsters were used in the experiments.

New "Alphatron" Vacuum Gauge.—A new "Alphatron" Vacuum Gauge, operating on a-c. current, is now offered by Naresco Equipment Corporation, equipment sales subsidiary of National Research Corporation. The Model 517 "Alphatron" Gauge also has these other features: (1) six ranges permitting the measurement of pressures from 1000 to 0.0001 mm. Hg on a linear scale; (2) instantaneous response, for quick readings; (3) compact construction and light weight; and (4) a connection for a recorder.

Like other NRC "Alphatron" vacuum gauges, the Model 517 gauge utilizes a sealed radium source; this emits alpha particles and produces ionized gas molecules which are collected on a plate to produce a current indicating directly the measured pressure. The gauge cannot be damaged by exposure to atmospheric pressure because the ion source operates at room temperature and at zero potential. "Alphatron" Vacuum Gauges are also less susceptible to contamination than are hot element vacuum gauges.

New Wide Range Hook-on Volt-Ammeter.—A new wide range pocket size volt-ammeter for measuring alternating current and voltage quickly and accurately has been introduced by the General Electric Company's Instrument Department.

Measuring only $9\frac{1}{8}$ by $3\frac{7}{16}$ by $1\frac{9}{16}$ in., the new product is designated AK-4, and includes 150/300/750 volts a.c. as its volt ranges.

Capable of measuring current on both insulated and non-insulated conductors without cutting the conductor or interrupting work, the new volt-ammeter is equipped with scales that automatically change when the range selector knob is turned, thus reducing the possibility of reading a wrong scale.

Current range of 10/30/100/300/800 amperes a.c. provides for the measurement of currents usually encountered in industrial or commercial circuits. In addition to the wide choice of ranges, a pointer stop mechanism has been incorporated contributing to the AK-4's versatility, the engineers said.

In measuring voltage the screw-in safety voltage leads are connected into recessed terminals for the protection of the operator. Then the desired voltage range and scale are selected by turning the selector knob.

Uses of the AK-4 include checking circuits, motor overloads, and determining motor starting and running currents. Ideal for maintenance personnel, electricians, and contractors, the device quickly diagnoses trouble without the need of shutting down equipment.

The new volt-ammeter operates on the frequency of 60 cycles with a 3 per cent full scale accuracy. Its hook-on assembly consists of a toroidal wound split-core current transformer with directionalized steel laminations. This current transformer is connected to the direct current indicating instrument through a shunting resistor circuit and rectifier.

Ultrasonic Beams for Bloodless Brain Surgery.—New possibilities in bloodless brain surgery were revealed at the American Physiological Society meeting in Madison, Wis., this fall. University of Illinois scientists told of using ultrasonic beams to destroy brain tissue in an area as small as 1/20 of an inch across—the width of the lead in a pencil—without affecting tissues around, above, or beyond the spot. Such precision offers possibilities far beyond the surgeon's knife for treatment of certain mental conditions. Sound beams with far less control already have been tried experimentally elsewhere for this. The University of Illinois device has not yet been used on human beings. A few further tests need to be made before the precision beam can be focused on any but animal subjects.

Sound from four high-energy sources built into one unit is focused at the point in the brain to be affected. While nerve tissue is killed, blood vessels are not affected.

Ultrasound is used at a frequency of a million cycles a second—50 times higher than the highest audible pitch. The intense beam is sent through a salt solution in contact with both sound sources and the brain. An opening is cut through the skull, but not through the protective membrane of the brain.

Any size or shape of area, at any depth in the brain, can be attacked. This precision in all directions may prove of value not only for treating mental disease, but also for study of the brain through learning exactly what areas control what functions.

Four years of work at the University of Illinois have gone into the device, reported by Prof. William J. Fry, Prof. John W. Barnard, and Rolfe F. Krumins in a paper titled "Effects of Intense Ultrasound on the Central Nervous System." Their work has been supported in part by the Office of Naval Research and the Air Force Aeromedical Laboratory.

New, Improved Sniperscope.—A sniperscope with a new look has been developed by the Corps of Engineers' Research and Development Laboratories, Fort Belvoir, Virginia.

An electronic infrared device designed to permit the soldier to sight and aim at a target in complete darkness without the aid of visible light, the sniperscope first made its appearance in World War II operations. It consists of a weapon-mounted infrared source and telescopic image tube, powered by a battery pack worn by the user. Invisible rays emanating from the infrared source are reflected back from objects in their path. They are picked up by the telescope above the weapon's sights where a visible image appears before the operator.

Now being issued to troops, the improved scope with its longer range, more rugged construction and more accurate aiming device eliminates many of the shortcomings of its predecessors, according to Mr. O. P. Cleaver, Chief of the Laboratories' Electrical Engineering Department.

Using the latest model, the infantryman can pinpoint a target in total darkness at greatly improved ranges. The increased range is due principally to the more powerful image tube which operates at 20,000 volts as compared with the 4,000 volts of the previous model.

More rugged construction and simpler operation of the new sniperscope almost halves the maintenance problems met in the old. Considering its delicate mechanism involving photoemissive tubes, lenses and sensitive construction, the device is even more rugged than could be expected. Complete field maintenance, which is now possible, will be facilitated by a new electronic maintenance shop now under test. Recharging the battery will be simplified by new charging kits under development.

Another improvement is the projected illuminated adjustable reticle, replacing the shadow type of the older model. This permits more accurate aiming.

The weight of the snipercope, 31½ pounds overall, still presents something of a problem. As more than one-half the weight is concentrated in the battery, power pack and light source, research is underway to develop smaller, more dependable batteries. Equipment is being tested which could eliminate the battery and light source as necessary parts to be carried by the soldier.

New Technique for Quality Control of Titanium.—Hydrogen content of titanium metal and alloys can be determined quickly and accurately by a new technique recently developed by National Research Corporation, Cambridge, Mass. Employing the NRC Model 912 Vacuum Fusion Gas Analysis Apparatus, this technique enables hydrogen content to be determined in less than 20 minutes. Accuracy is better than plus or minus 4 per cent.

This is of importance because recent work has established that small concentrations of hydrogen in titanium metals and alloys may have critical and adverse effects on their physical properties. This new technique for the rapid analysis of hydrogen content can be used to eliminate costly work on defective titanium or titanium alloys.